From Perceptual Model Of Visualization To Visual-Centric Computation

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Luc Legay
Label your
Professional Networks
Social media
Medias, comm, contenus
Divers
Explorateurs éclaireurs
Les historiques
Fing, institutions, recherch
Experts & consultants
Marketing, eMarketing

From Perceptual Model Of Visualization To Visual-Centric Computation

Big data

- Computation of big data is extremely expensive
- Visualization of the result is not understandable

From Perceptual Model Of Visualization
To Visual-Centric Computation

Visual-centric computation

- To speed up the result generating process
- Improve the understandability of the visualization

From Perceptual Model Of Visualization To Visual-Centric Computation

Visual-centric computation

- How close the result is to the optimal solution in the data →
- **How close the visualization is to the maximum in the user's visual ability**

- Algorithm(data, parameters) →
- **Algorithm(data, parameters, visual limitations)**
  - Visual limitations := limitations in the screen resolution and the human visual system.

Overview

1. Perceptual Model of Visualization
   - Quantify visual limitations
2. Visual Feature
   - Generalize the perceptual model of visualization
3. Visual-Centric Computation
   - Use visual limitations to guide computation

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1

2

Overview

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Overview

1. ☑ Perceptual Model of Visualization
2. ⇥ Visual Feature
3. ☐ Visual-Centric Computation

Overview

1. Perceptual Model of Visualization
   - Quantify visual limitations using perception law
2. Visual Feature (On-going)
   - Generalize perceptual models of visualization
3. Visual-Centric Computation (Future Work)
   - Use visual limitations to guide computation

Perceptual Model of Visualization

- Perceptual Model for correlation
- IEEE VIS paper, 2014
- Harrison, Lane, **Fumeng Yang**, Steven Franconeri, and Remco Chang. "**Ranking Visualizations of Correlation Using Weber's Law**." IEEE Transactions on Visualization and Computer Graphics (2014): 1

1 Perceptual Model of Visualization

1.1 Context

- Use a visualization properly
  - Understand the **perception** of the visualization
  - **Run perceptual experiments**

1.1 Context

- Classical way, A – B test
  - Visualization A is better than the other B in some cases
  - Setting A of a visualization is better than the other B in some cases
  - …

1.1 Context

- Classical way, A – B test
  - Visualization A is better than the other B in some cases
  - Setting A of a visualization is better than the other B in some cases
  - …
- Only effects were identified

1.1 Context

- Issues
- No further underlying implication
  - Hard to apply to design
- Hard to Scale
  - For comparing visualizations, pairwise comparison
  - 9 visualizations = C(9, 2) = 36 experiments

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  - 9 visualizations = C(9, 2) = 36 experiments

1 Perceptual Model of Visualization

- 1.1 Context
- 1.2 Contribution
- 1.3 Related Work
- 1.4 Experiment
- 1.5 Implication
- 1.6 Summary

1.2 Contribution

- Model Driven Approach
  - Build models for the perception of correlation in 9 visualizations
  - Analyze the perception of visualizations using models

1.2 Contribution

- Model Driven Approach
  - A step forward to evaluate visualization
    - Don’t have to run A-B test and do pairwise comparison
  - Not only effects were identified
  - Wider applicable range of findings

1.2 Contribution

- Model Driven Approach
  - A classical perceptual law -- Weber's law -- holds for the perception of correlation in 9 visualizations

1 Perceptual Model of Visualization

- 1.1 Context
- 1.2 Contribution
- 1.3 Related Work
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1.3 Related Work

- Weber’s Law
- Ernst Heinrich Weber (1795–1878)

1.3 Related Work

- Weber's Law
- Ernst Heinrich Weber (1795–1878)
- One of the first people to approach the study of the human response to a physical stimulus in a quantitative fashion*

- Historically important psychological law
  - **quantifying** the perception of change in a given stimulus
- The law is the starting of quantitative psychology**

* Ross, H.E. and Murray, D. J.(1996)(Ed. and Transl.) E.H.Weber on the tactile senses. 2nd ed. Hove: Erlbaum (UK) Taylor & Francis.
** Hoagland, Hudson. "The Weber-Fechner law and the all-or-none theory." The Journal of General Psychology 3.3 (1930): 351-373.

1.3 Related Work

- Weber's Law
- Perceptual Law for brightness, length etc.
- The discrimination threshold of two stimuli is proportional to the intensity of the stimulus

1.3 Related Work

- Weber's Law
- $dp = k \cdot dS / S$
    - dp, the change in perception
    - dS, the differential increase in the stimulus
    - S is the intensity of the stimulus
    - k is the coefficient
- To get one unit change in perception, the change in physical stimulus is proportional to the intensity of the stimulus
- Just Noticeable Difference (JND)

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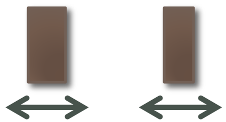
1.3 Related Work

- Weber’s Law



1.3 Related Work

- Weber's Law

0.2″ v.s. 0.15″

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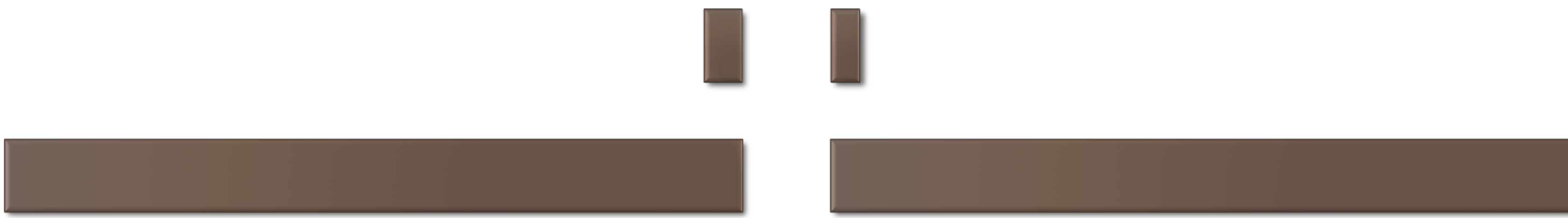
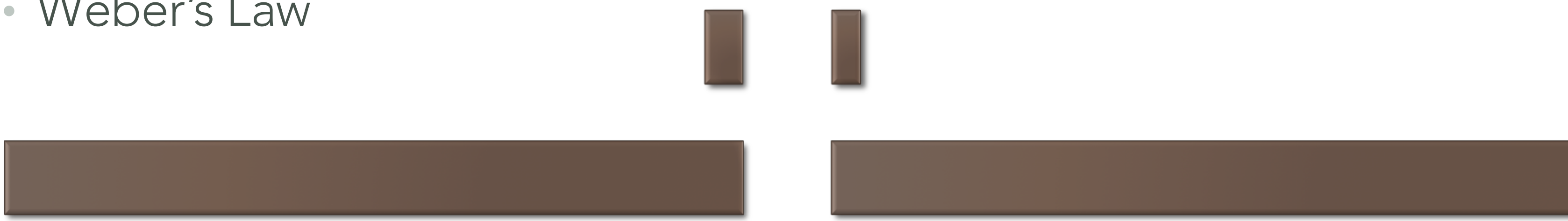


4.1″ v.s. 4.15″

1.3 Related Work

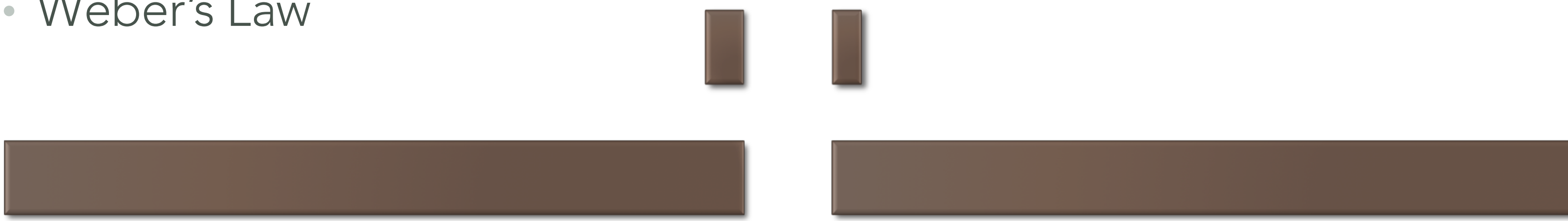
- Weber's Law



- Same amount of difference, Δ = 0.05″
- JND top < 0.05″
- JND bottom > 0.05″

1.3 Related Work

- Weber's Law



- JND is linear to the intensity of the stimulus (i.e. length)
- JND = k * length + b

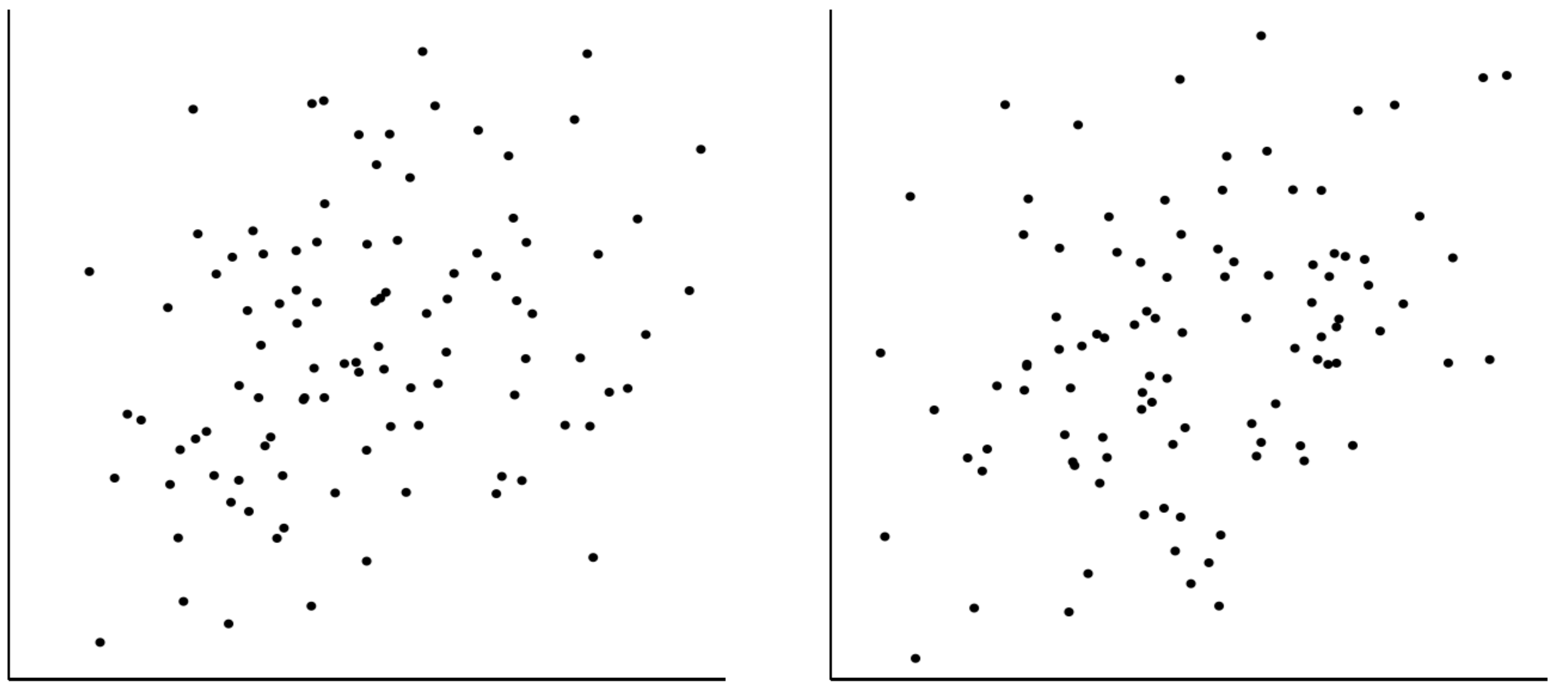
1.3 Related Work

- Previous work*
- The perception of correlation in scatterplots could be modeled using Weber's law

* Rensink, Ronald A., and Gideon Baldridge. "The perception of correlation in scatterplots." Computer Graphics Forum. Vol. 29. No. 3. Blackwell Publishing Ltd, 2010.

1.3 Related Work

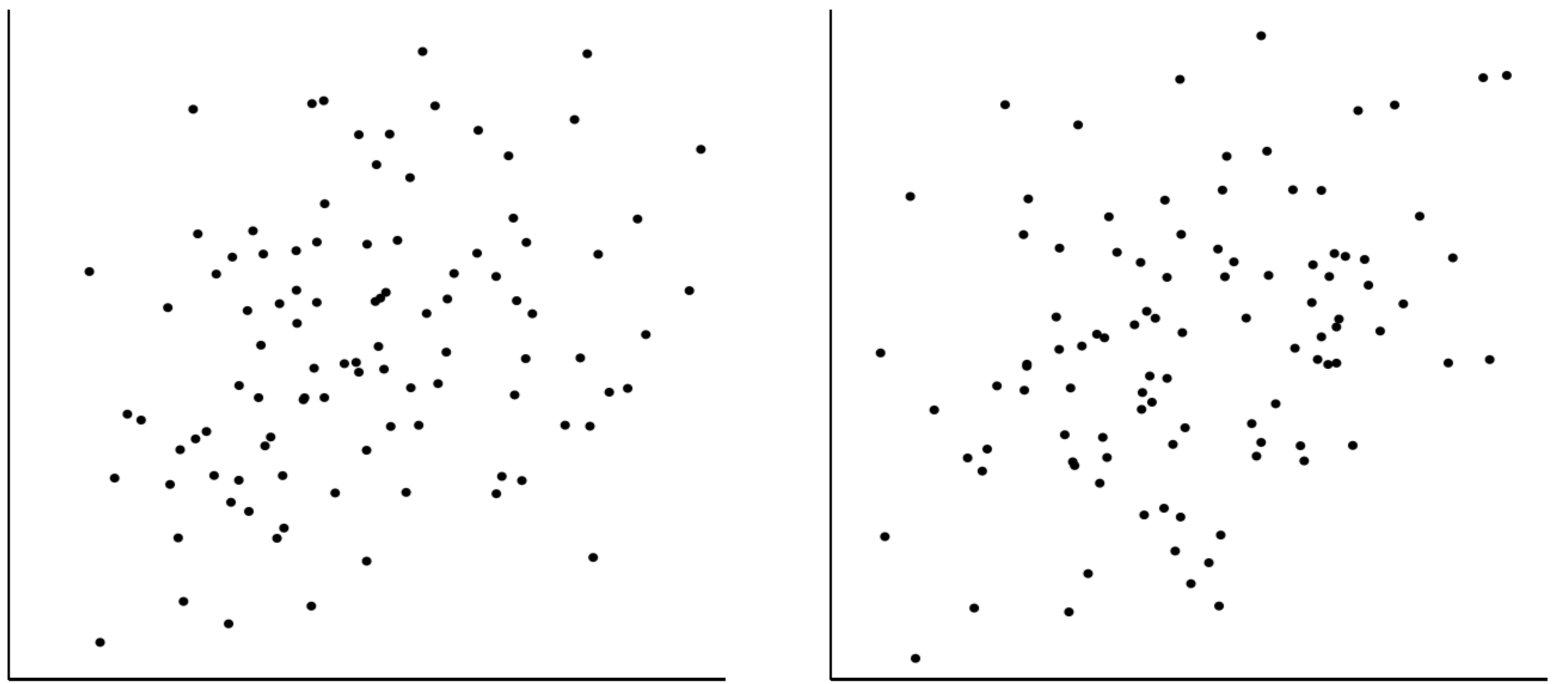
- Weber's Law for correlation



1.3 Related Work

- Weber's Law for correlation

$r = 0.3$ $r = 0.35$



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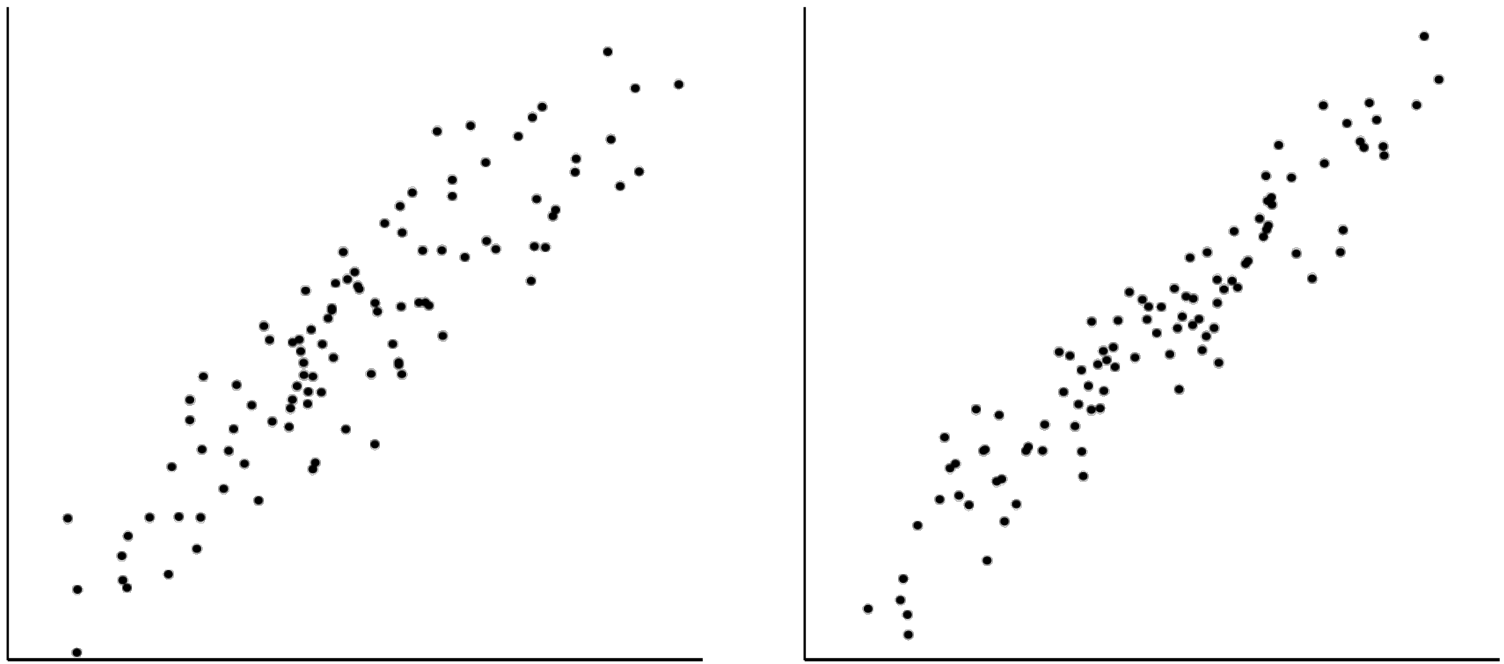

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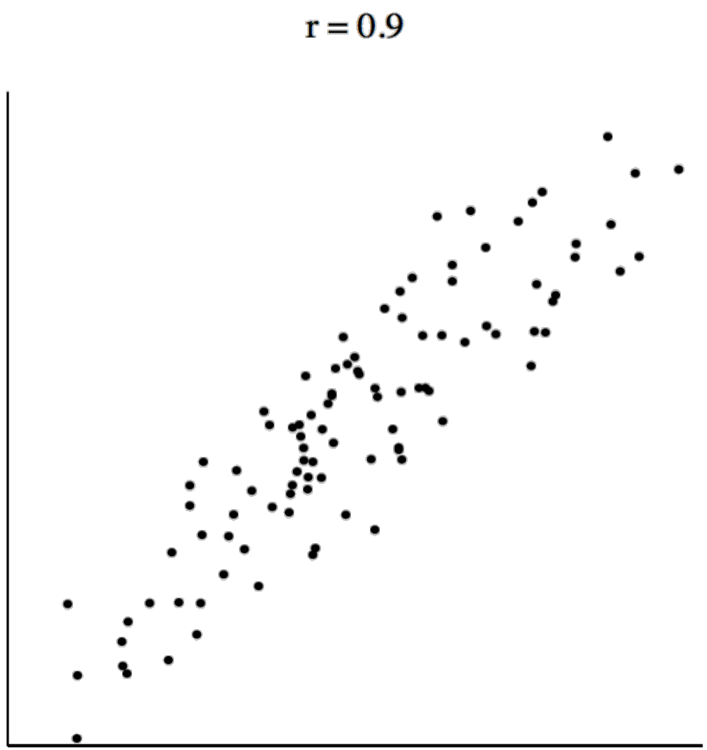
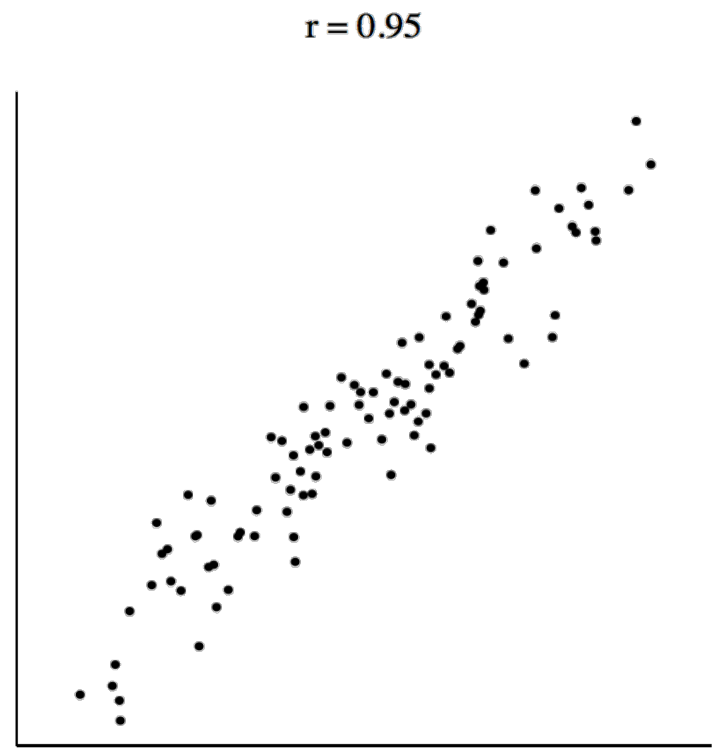




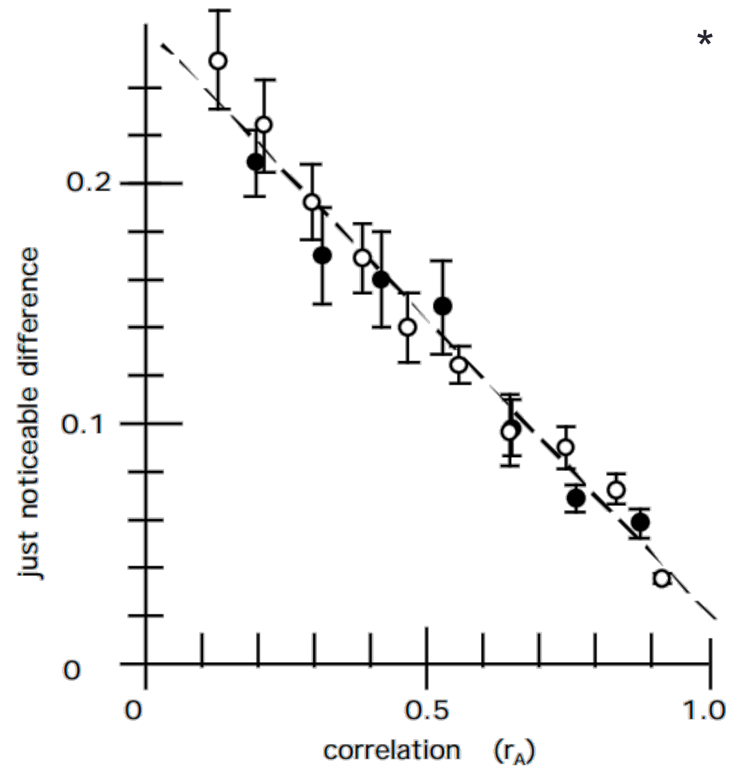
1.3 Related Work

- Weber’s Law for correlation

- Same amount of difference, $\Delta r = 0.05$
- JND of $r = 0.3$, > 0.05
- JND of $r = 0.9$, < 0.05

1.3 Related Work

- Weber's Law for correlation



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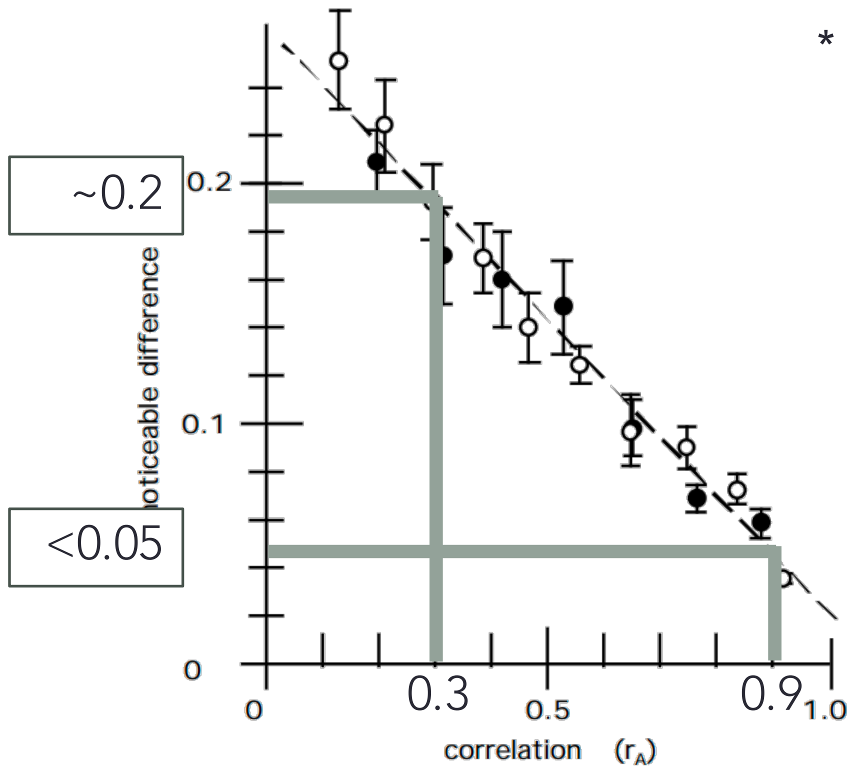

* Rensink, Ronald A., and Gideon Baldridge. "The perception of correlation in scatterplots." Computer Graphics Forum. Vol. 29. No. 3. Blackwell Publishing Ltd, 2010.

1.3 Related Work

- We followed this previous work

- Build models for the perception of other visualizations on correlation
- Analyze the perception of correlation in visualizations using the **models**

1 Perceptual Model of Visualization

- 1.1 Context
- 1.2 Contribution
- 1.3 Related Work
- 1.4 Experiment
- 1.5 Implication
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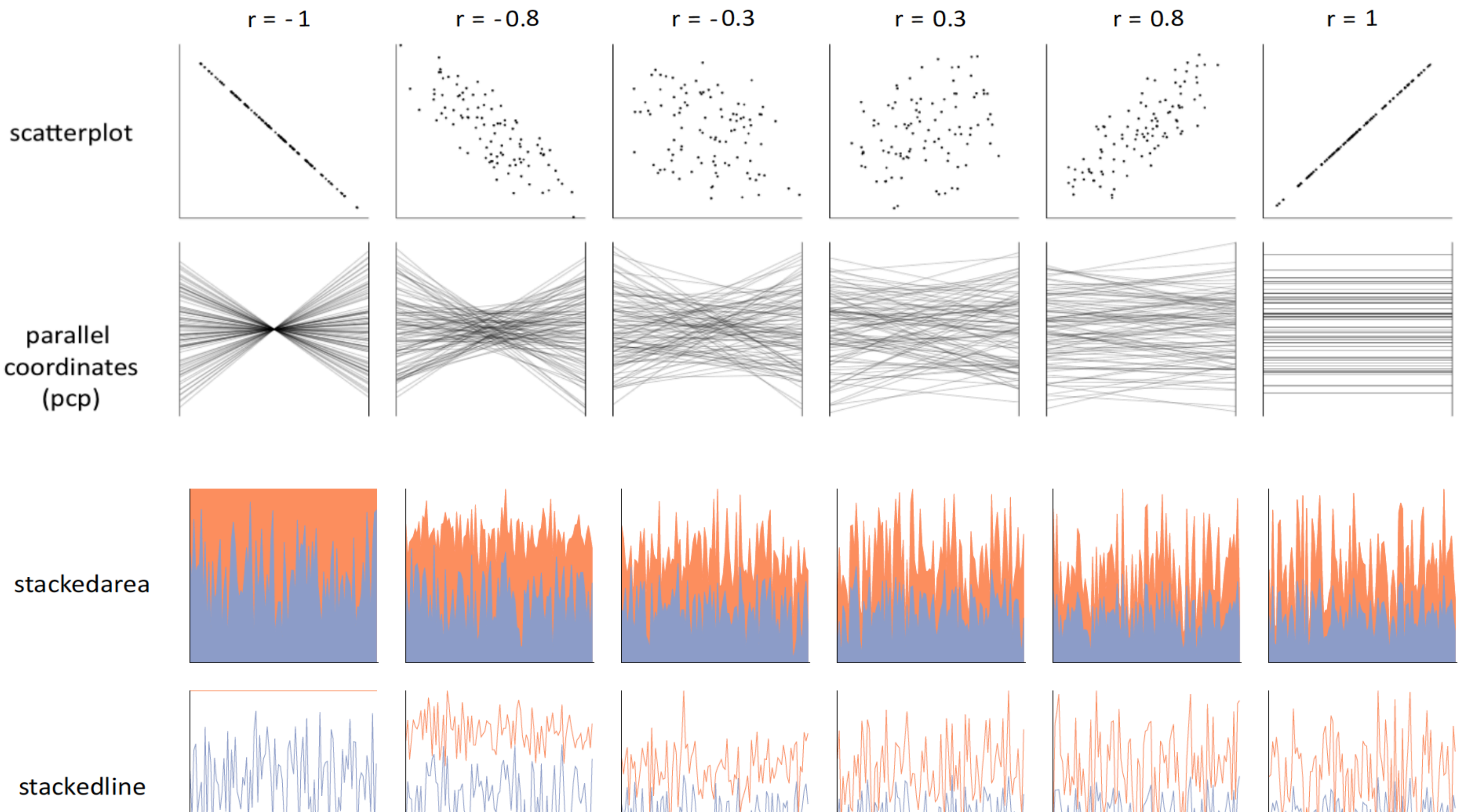
1.4 Experiment

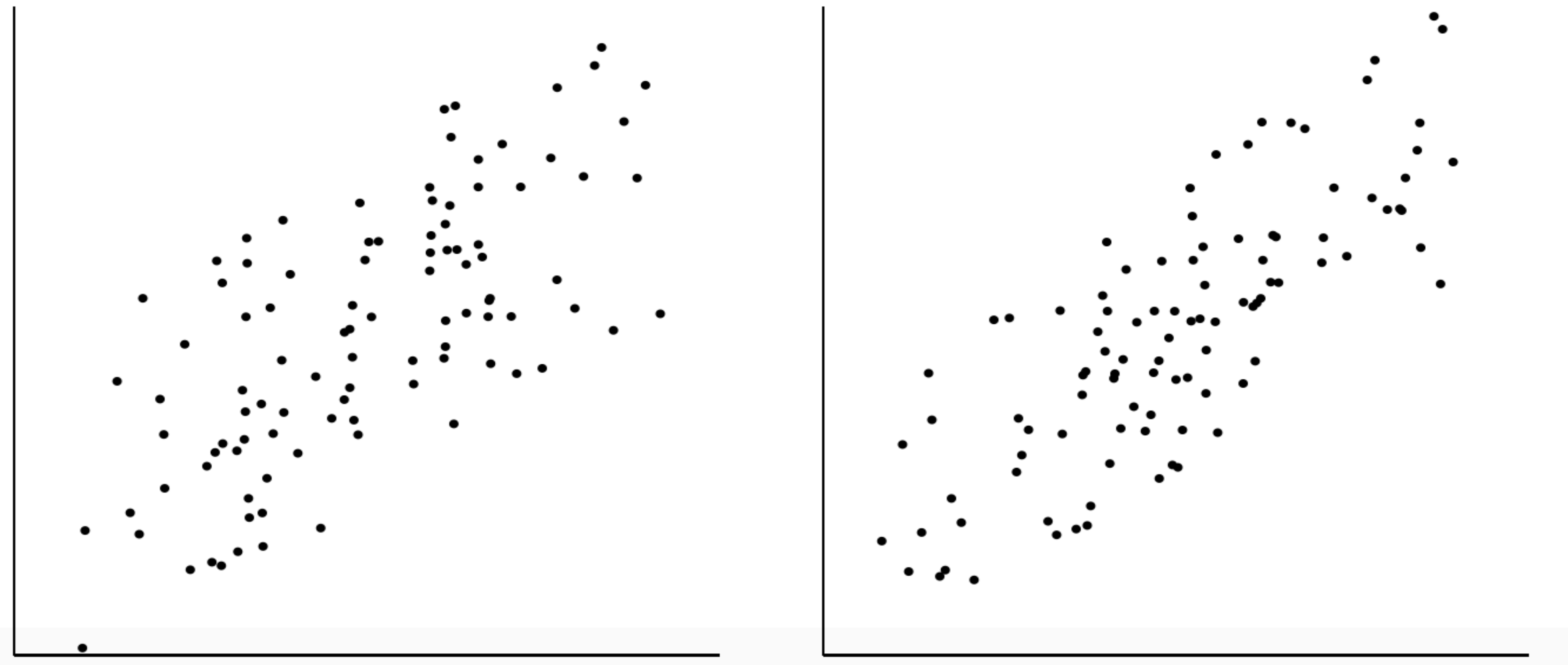
- 9 visualizations
  - On bivariate data for correlation
  - Commonness

1.4 Experiment



1.4 Experiment

- Discrimination Task

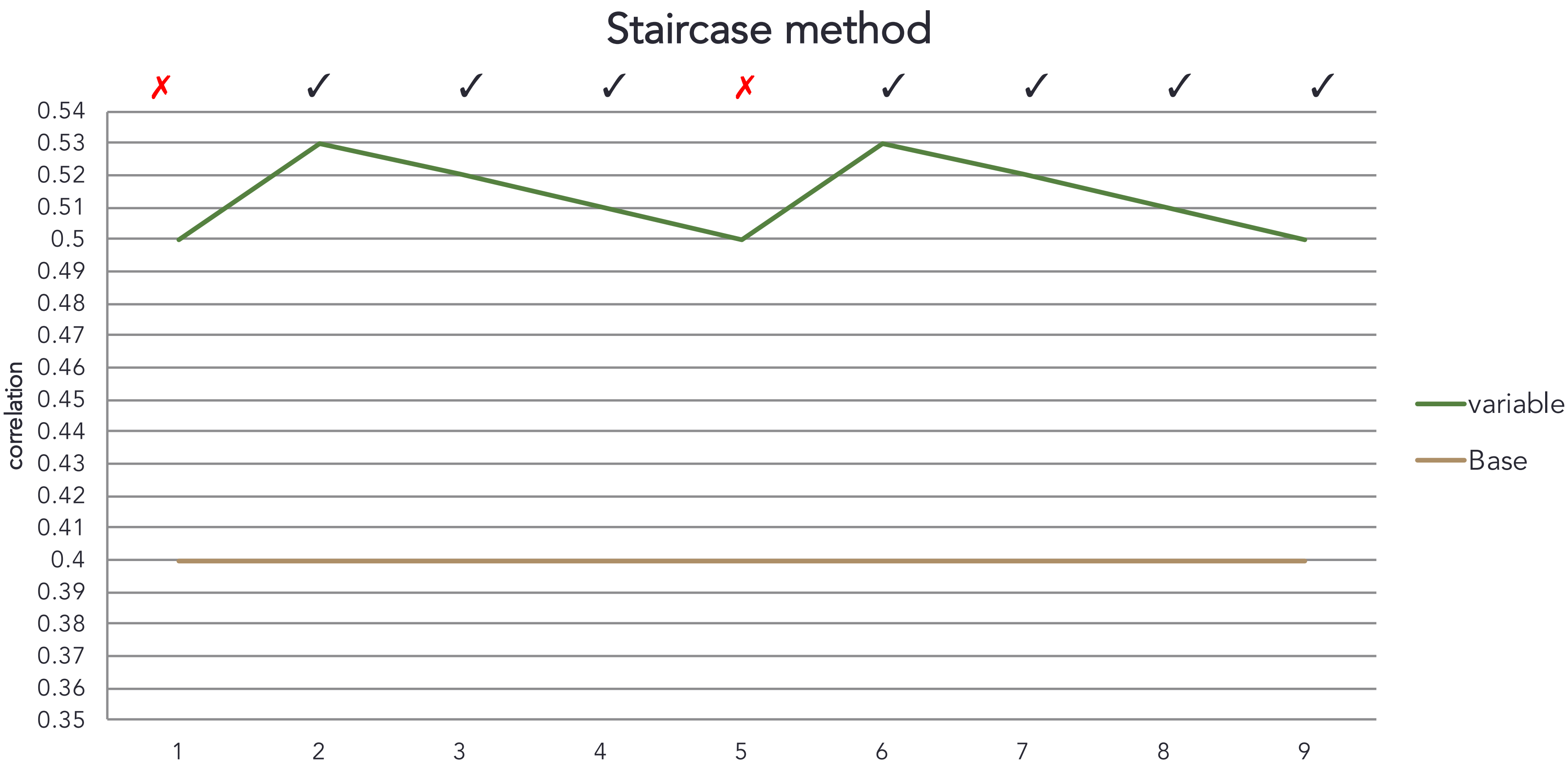


- Which one is more correlated?

1.4 Experiment

- Staircase method
  - Adjust the difference between two plots based on the judgment correctness

1.4 Experiment



1.4 Experiment

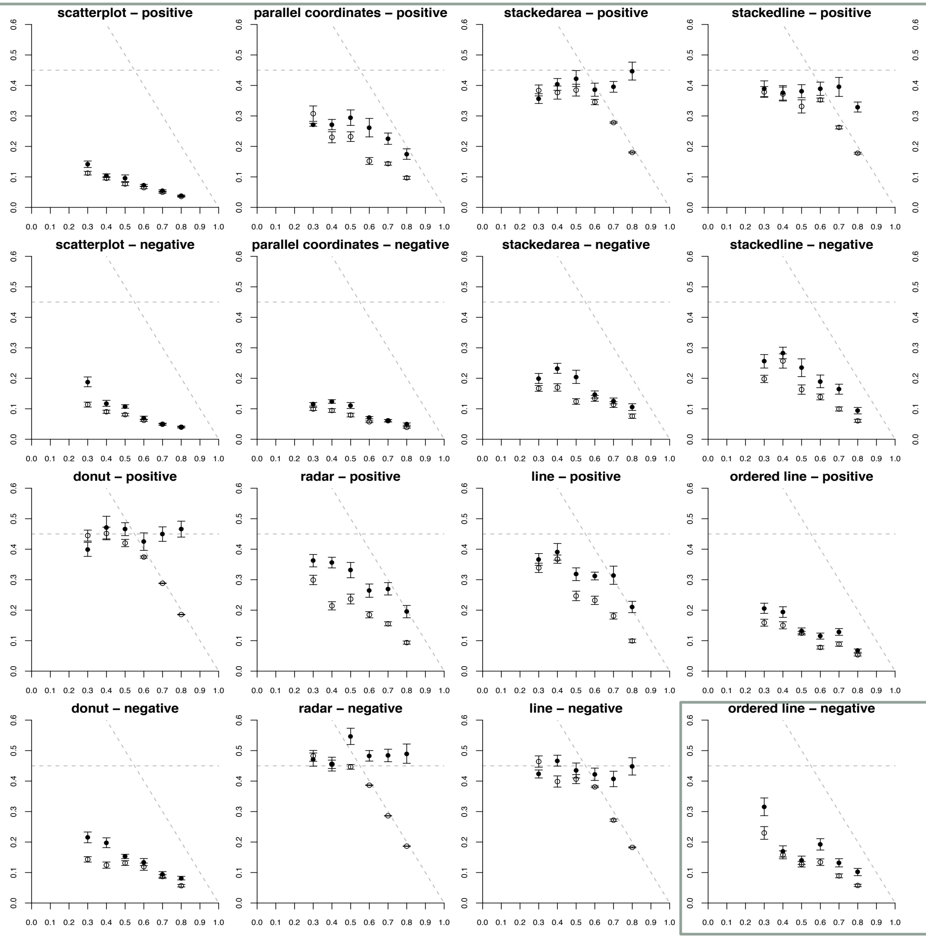
- Terminate when the answers in recent judgments are consistent or 50 judgments
- JND of correlation = Average of the difference between two plots in recent judgments

1.4 Experiment

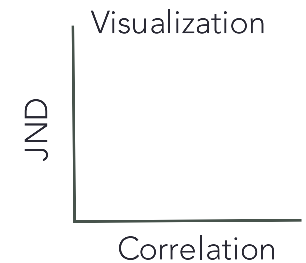
- 9 visualizations, scatterplots, parallel coordinates etc.
- 2 types of data, positive and negative correlated dataset
- 6 base cases, r = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8

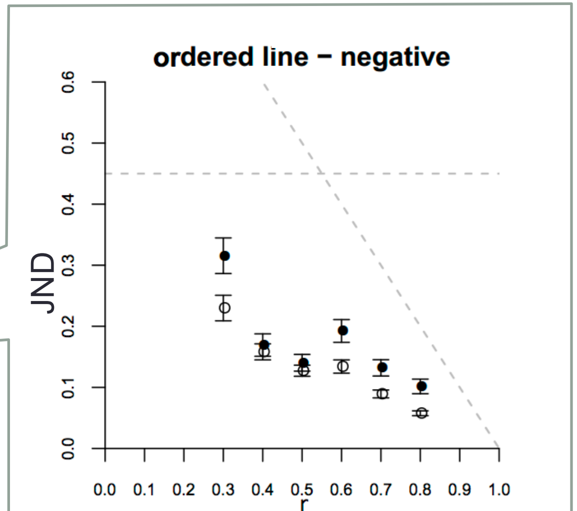
- 9 x 2 x 6 = 108 conditions

1.4 Experiment



Results





1.4 Experiment

- Data
  - JND, correlation, visualizations
  - positive and negative correlated datasets
- Analyze data
  - Statistics test
  - Models fit

1.4 Experiment

- Analyze data
  - Statistics test
  - Models fit

1.4 Experiment

- If there is significant difference between visualizations?
- Not normally distributed
  - Non-parameteric
  - Kruskal-Wallis
    - If there is significant difference
    - p-value < 0.05 → Yes!
  - Mann-Whitney, post hoc test
    - Where is the difference, which two are different
    - Bonferonni correction ($p < 0.0036$)
    - → next page

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1.4 Experiment

- Significance between charts?

visualization - direction 1	visualization - direction 2	W	p-value
scatterplot - negative	scatterplot - positive	51165.5	0.54
scatterplot - negative	parallel coordinates - positive	10885.5	$< 0.001^*$
scatterplot - positive	parallel coordinates - positive	8623	$< 0.001^*$
parallel coordinates - negative	scatterplot - negative	51291	0.42
parallel coordinates - negative	scatterplot - positive	51491	0.16
parallel coordinates - negative	parallel coordinates - positive	8641.5	$< 0.001^*$
stacked bar - negative	stacked line - negative	34421	$< 0.001^*$
stacked bar - negative	stacked area - negative	33348.5	$< 0.001^*$
stacked bar - negative	donut - negative	43361	0.037
stacked line - negative	stacked area - negative	66646	0.014
line - positive	radar - positive	73775.5	0.0017^*
line - positive	ordered line - positive	104163.5	$< 0.001^*$
line - positive	ordered line - negative	101883	$< 0.001^*$
ordered line - negative	ordered line - positive	66292	0.0075

1.4 Experiment

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1.4 Experiment

- Analyze data
  - Statistics test → difference, effects
  - Models fit

1.4 Experiment

- Analyze data
  - Statistics test → difference, effects
  - Models fit → model the perception of visualization

1.4 Experiment

- Model Driven Approach
- Regression
  - JND and intensity of correlation
  - for each visualization

1.4 Experiment

- Linear model fits well for JND and correlation r

visualization - direction	intercept-b	slope-k	correlation-r	r^2	RMS
scatterplot - positive	0.17	-0.17	-0.99	0.98	0.0041
scatterplot - negative	0.21	-0.22	-0.95	0.90	0.013
parallel coordinates - positive	0.37	-0.27	-0.86	0.74	0.032
parallel coordinates - negative	0.16	-0.14	-0.95	0.90	0.0085
stacked line - negative	0.35	-0.32	-0.92	0.84	0.027
stacked area - negative	0.27	-0.22	-0.93	0.86	0.016
stacked bar - negative	0.22	-0.19	-0.95	0.90	0.011
donut - negative	0.26	-0.23	-0.96	0.93	0.012
line - positive	0.46	-0.32	-0.86	0.74	0.043
radar - positive	0.44	-0.36	-0.95	0.91	0.024
ordered line - positive	0.26	-0.24	-0.95	0.91	0.014
ordered line - negative	0.32	-0.31	-0.88	0.78	0.031

1.4 Experiment

Model fits very well

visualization - direction	intercept-b	slope-k	correlation-r	r^2	RMS
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1.4 Experiment

Parameters of the model

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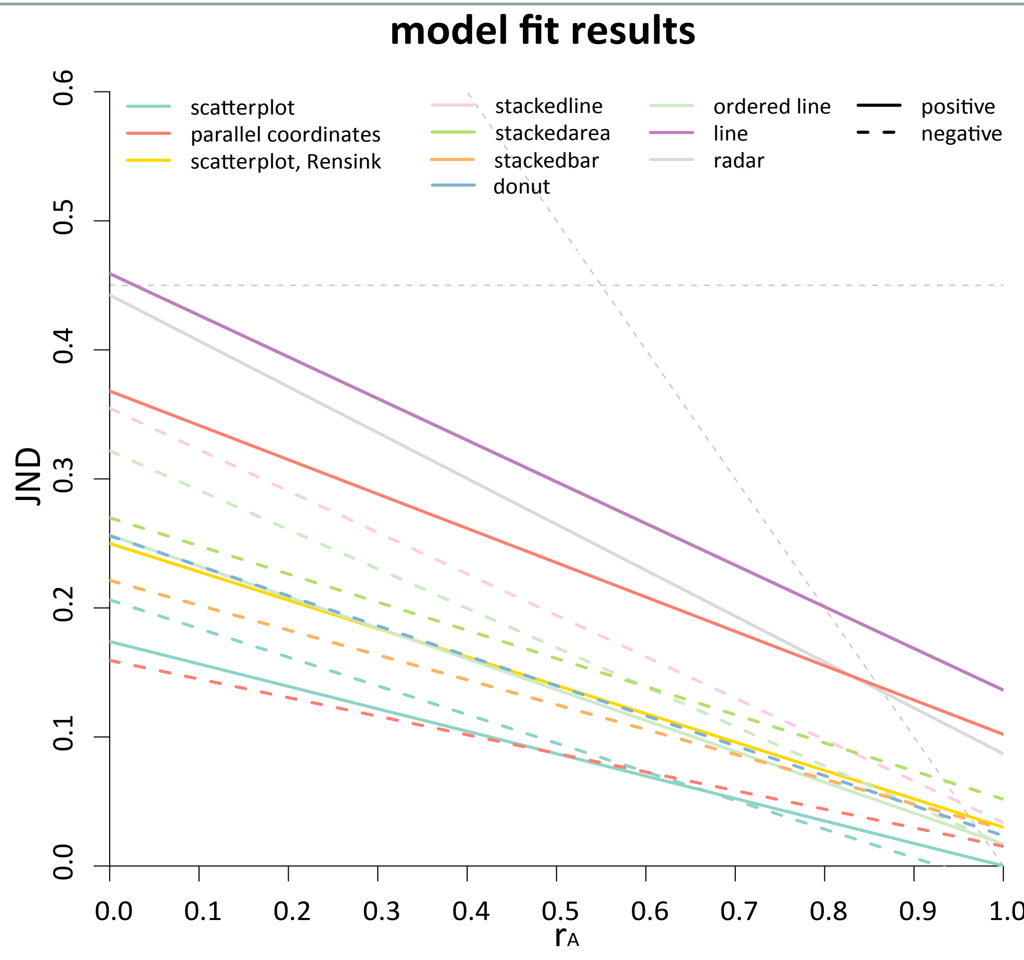
1.4 Experiment



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1.5 Implication

- Contribution

1.5 Implication

- Linear model fits for JND and correlation for all nine visualizations

1.5 Implication

- Linear model fits for JND and correlation for all nine visualizations

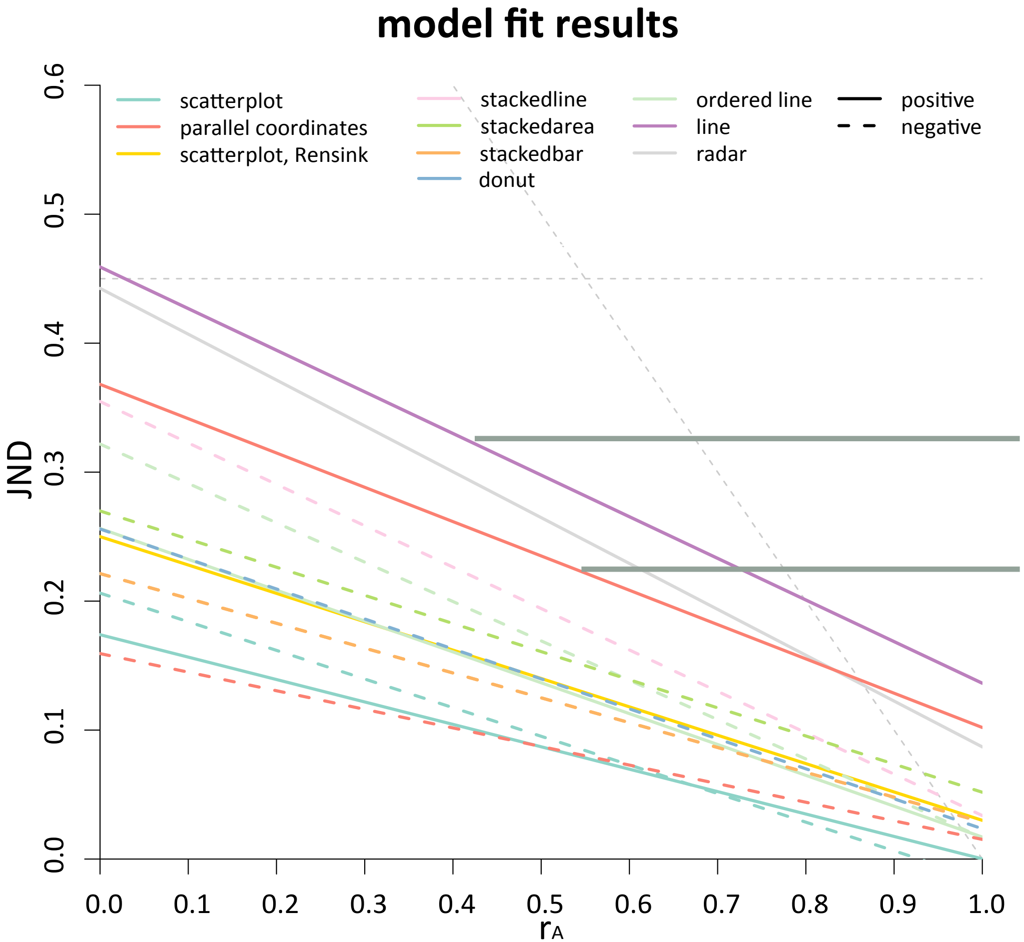
- Weber's law holds for these nine visualizations on the perception of correlation

1.5 Implication

- Perception of correlation in these visualizations is known inside of the range of these models

- Analyze the perception using the same "language"
  - Look at the models → infer more information

1.5 Implication

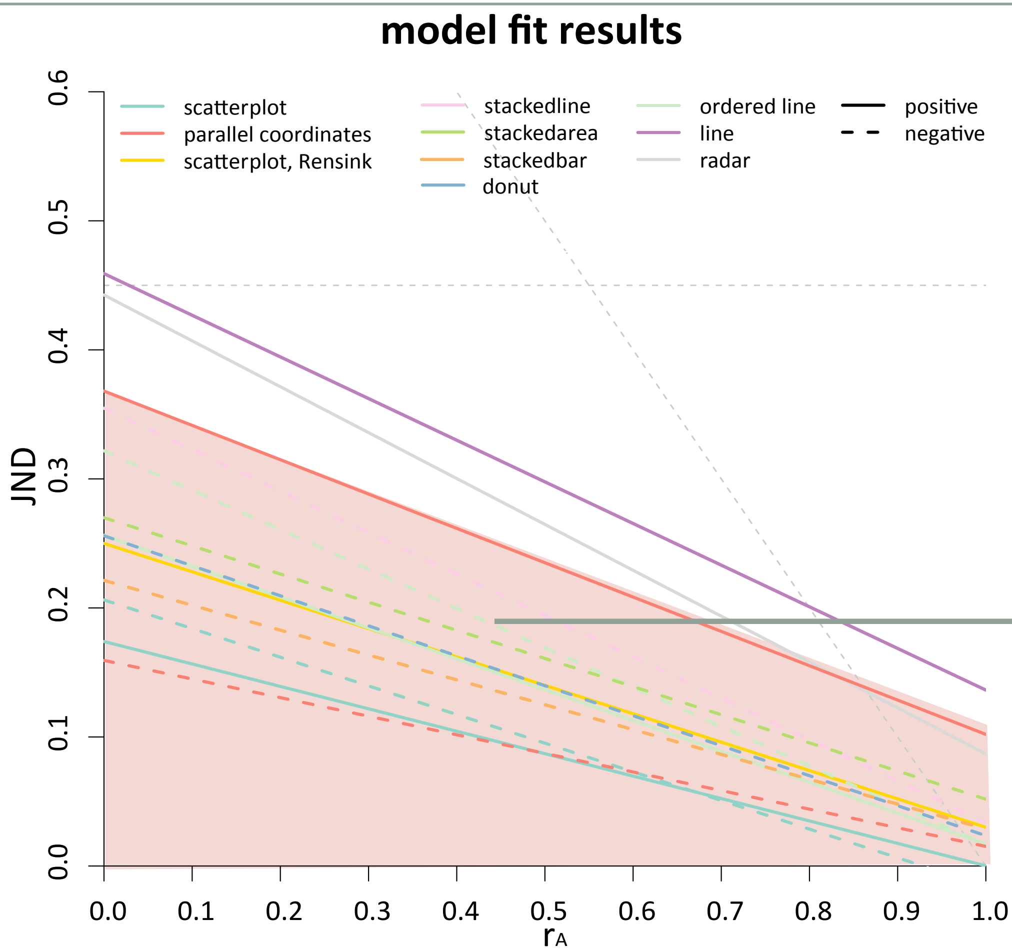


line chart for positive

parallel coordinates for positive

better

1.5 Implication



Integral
overall performance

1.5 Implication

- Ranking Table

r = 0.1 *	r = 0.3	r = 0.5	r = 0.7	r = 0.9 *	overall
pcp−negative	pcp−negative	scatterplot−positive	scatterplot−negative	scatterplot−negative	scatterplot−positive
scatterplot−positive	scatterplot−positive	pcp−negative	scatterplot−positive	scatterplot−positive	pcp−negative
scatterplot−negative	scatterplot−negative	scatterplot−negative	pcp−negative	pcp−negative	scatterplot−negative
stackedbar−negative	stackedbar−negative	stackedbar−negative	stackedbar−negative	ordered line−positive	stackedbar−negative
ordered line−positive	ordered line−positive	ordered line−positive	ordered line−positive	donut−negative	ordered line−positive
donut−negative	donut−negative	donut−negative	donut−negative	ordered line−negative	donut−negative
stackedarea−negative	stackedarea−negative	stackedarea−negative	ordered line−negative	stackedbar−negative	stackedarea−negative
ordered line−negative	ordered line−negative	ordered line−negative	stackedarea−negative	stackedline−negative	ordered line−negative
stackedline−negative	stackedline−negative	stackedline−negative	stackedline−negative	stackedarea−negative	stackedline−negative
pcp−positive	pcp−positive	pcp−positive	pcp−positive	radar−positive	pcp−positive
radar−positive	radar−positive	radar−positive	radar−positive	pcp−positive	radar−positive
line−positive	line−positive	line−positive	line−positive	line−positive	line−positive

better

1.7 Implication

- Ranking Table

Predicted

better

r = 0.1 *	r = 0.3	r = 0.5	r = 0.7	r = 0.9 *	overall
pcp−negative	pcp−negative	scatterplot−positive	scatterplot−negative	scatterplot−negative	scatterplot−positive
scatterplot−positive	scatterplot−positive	pcp−negative	scatterplot−positive	scatterplot−positive	pcp−negative
scatterplot−negative	scatterplot−negative	scatterplot−negative	pcp−negative	pcp−negative	scatterplot−negative
stackedbar−negative	stackedbar−negative	stackedbar−negative	stackedbar−negative	ordered line−positive	stackedbar−negative
ordered line−positive	ordered line−positive	ordered line−positive	ordered line−positive	donut−negative	ordered line−positive
donut−negative	donut−negative	donut−negative	donut−negative	ordered line−negative	donut−negative
stackedarea−negative	stackedarea−negative	stackedarea−negative	ordered line−negative	stackedbar−negative	stackedarea−negative
ordered line−negative	ordered line−negative	ordered line−negative	stackedarea−negative	stackedline−negative	ordered line−negative
stackedline−negative	stackedline−negative	stackedline−negative	stackedline−negative	stackedarea−negative	stackedline−negative
pcp−positive	pcp−positive	pcp−positive	pcp−positive	radar−positive	pcp−positive
radar−positive	radar−positive	radar−positive	radar−positive	pcp−positive	radar−positive
line−positive	line−positive	line−positive	line−positive	line−positive	line−positive

1.5 Implication

- Ranking Table

Best

better

r = 0.1 *	r = 0.3	r = 0.5	r = 0.7	r = 0.9 *	overall
pcp−negative	pcp−negative	scatterplot−positive	scatterplot−negative	scatterplot−negative	scatterplot−positive
scatterplot−positive	scatterplot−positive	pcp−negative	scatterplot−positive	scatterplot−positive	pcp−negative
scatterplot−negative	scatterplot−negative	scatterplot−negative	pcp−negative	pcp−negative	scatterplot−negative
stackedbar−negative	stackedbar−negative	stackedbar−negative	stackedbar−negative	ordered line−positive	stackedbar−negative
ordered line−positive	ordered line−positive	ordered line−positive	ordered line−positive	donut−negative	ordered line−positive
donut−negative	donut−negative	donut−negative	donut−negative	ordered line−negative	donut−negative
stackedarea−negative	stackedarea−negative	stackedarea−negative	ordered line−negative	stackedbar−negative	stackedarea−negative
ordered line−negative	ordered line−negative	ordered line−negative	stackedarea−negative	stackedline−negative	ordered line−negative
stackedline−negative	stackedline−negative	stackedline−negative	stackedline−negative	stackedarea−negative	stackedline−negative
pcp−positive	pcp−positive	pcp−positive	pcp−positive	radar−positive	pcp−positive
radar−positive	radar−positive	radar−positive	radar−positive	pcp−positive	radar−positive
line−positive	line−positive	line−positive	line−positive	line−positive	line−positive

1.5 Implication

- Ranking Table

better

r = 0.1 *	r = 0.3	r = 0.5	r = 0.7	r = 0.9 *	overall
pcp−negative	pcp−negative	scatterplot−positive	scatterplot−negative	scatterplot−negative	scatterplot−positive
scatterplot−positive	scatterplot−positive	pcp−negative	scatterplot−positive	scatterplot−positive	pcp−negative
scatterplot−negative	scatterplot−negative	scatterplot−negative	pcp−negative	pcp−negative	scatterplot−negative
stackedbar−negative	stackedbar−negative	stackedbar−negative	stackedbar−negative	ordered line−positive	stackedbar−negative
ordered line−positive	ordered line−positive	ordered line−positive	ordered line−positive	donut−negative	ordered line−positive
donut−negative	donut−negative	donut−negative	donut−negative	ordered line−negative	donut−negative
stackedarea−negative	stackedarea−negative	stackedarea−negative	ordered line−negative	stackedbar−negative	stackedarea−negative
ordered line−negative	ordered line−negative	ordered line−negative	stackedarea−negative	stackedline−negative	ordered line−negative
stackedline−negative	stackedline−negative	stackedline−negative	stackedline−negative	stackedarea−negative	stackedline−negative
pcp−positive	pcp−positive	pcp−positive	pcp−positive	radar−positive	pcp−positive
radar−positive	radar−positive	radar−positive	radar−positive	pcp−positive	radar−positive
line−positive	line−positive	line−positive	line−positive	line−positive	line−positive

Not good

1 Perceptual Model of Visualization

1.6 Summary

- Perceptual model of visualization
    - Harrison, Lane, **Fumeng Yang**, Steven Franconeri, and Remco Chang. "Ranking Visualizations of Correlation Using Weber's Law." IEEE Transactions on Visualization and Computer Graphics (2014): 1.
- Weber's law holds for perception of correlation on 9 visualizations
- Compare 9 visualizations using Weber models

Overview

1. Perceptual Model of Visualization
   - Quantify visual limitations using perception law
2. Visual Feature (On-going)
   - Generalize perceptual models of visualization
3. Visual-Centric Computation (Future Work)
   - Use visual limitations to guide computation

2 Visual Feature

- 2.1 Contribution
- 2.2 Hypothesis
- 2.3 Test Hypothesis
- 2.4 Implication
- 2.5 Summary

2.1 Contribution

- **Weber's** law holds for the perception of **correlation** in nine visualizations
- … Don't know why

2.1 Contribution

- Contribution

2.1 Contribution

- Why does the Weber’s law work for correlation?

2.1 Contribution

- Why does the Weber's law work for correlation?

2.1 Contribution

- Why does the Weber's law work for correlation?
  - A perceptual law for length and brightness works for a statistical measurement, correlation
  - When the Weber model works and when not, without exhaustively testing all cases?

2 Visual Feature

2.2 Hypothesis

- Intuition
- Instead of **judging correlation** in their brains, participants are using something as the **substitute**

2.2 Hypothesis

- Intuition
- Instead of **judging correlation** in their brains, participants are using something as the **substitute**
- 3 evidence

2.2 Hypothesis

- Evidence 1
- Participants could finish perceiving correlation in 100ms*
    - 100ms are enough to see but not compute nor think

* Rensink, Ronald A. "On the Prospects for a Science of Visualization." Handbook of Human Centric Visualization. Springer New York, 2014. 147-175.

2.2 Hypothesis

- Evidence 1
- Participants could finish perceiving correlation in 100ms*
  - Brain study
  - 100ms, the stimulus is still in primary visual cortex
  - Primary visual cortex = global feature and edge information

* Rensink, Ronald A. "On the Prospects for a Science of Visualization." Handbook of Human Centric Visualization. Springer New York, 2014. 147-175.

2.2 Hypothesis

- Evidence 2
- Comments from the participants
- What's your visual strategy?
  - "I looked at the length and the width of all of the dots compiled."
  - "I looked to see which ones were farthest away from the center."
  - ...Suggest that participants are perceiving some features.

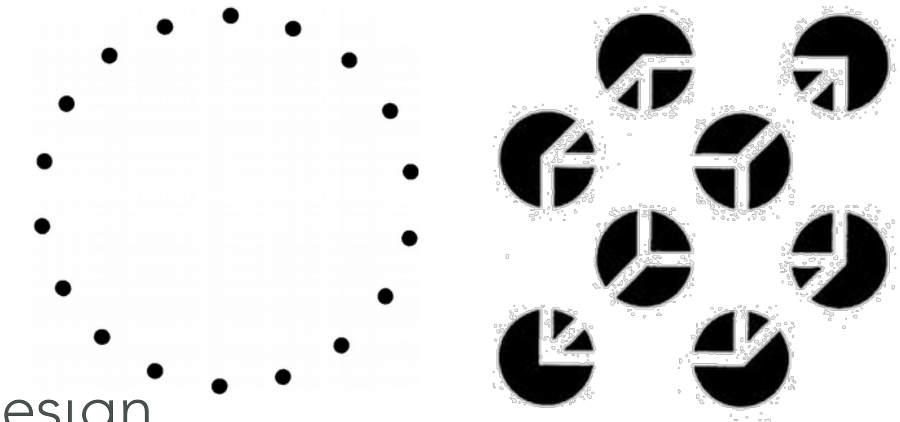
2.2 Hypothesis

- Evidence 2
- Comments from the participants
- What’s your visual strategy?

  - “I looked at the length and the width of all of the dots compiled.”
  - “I looked to see which ones were farthest away from the center.”
  - …Suggest that participants are perceiving some features.

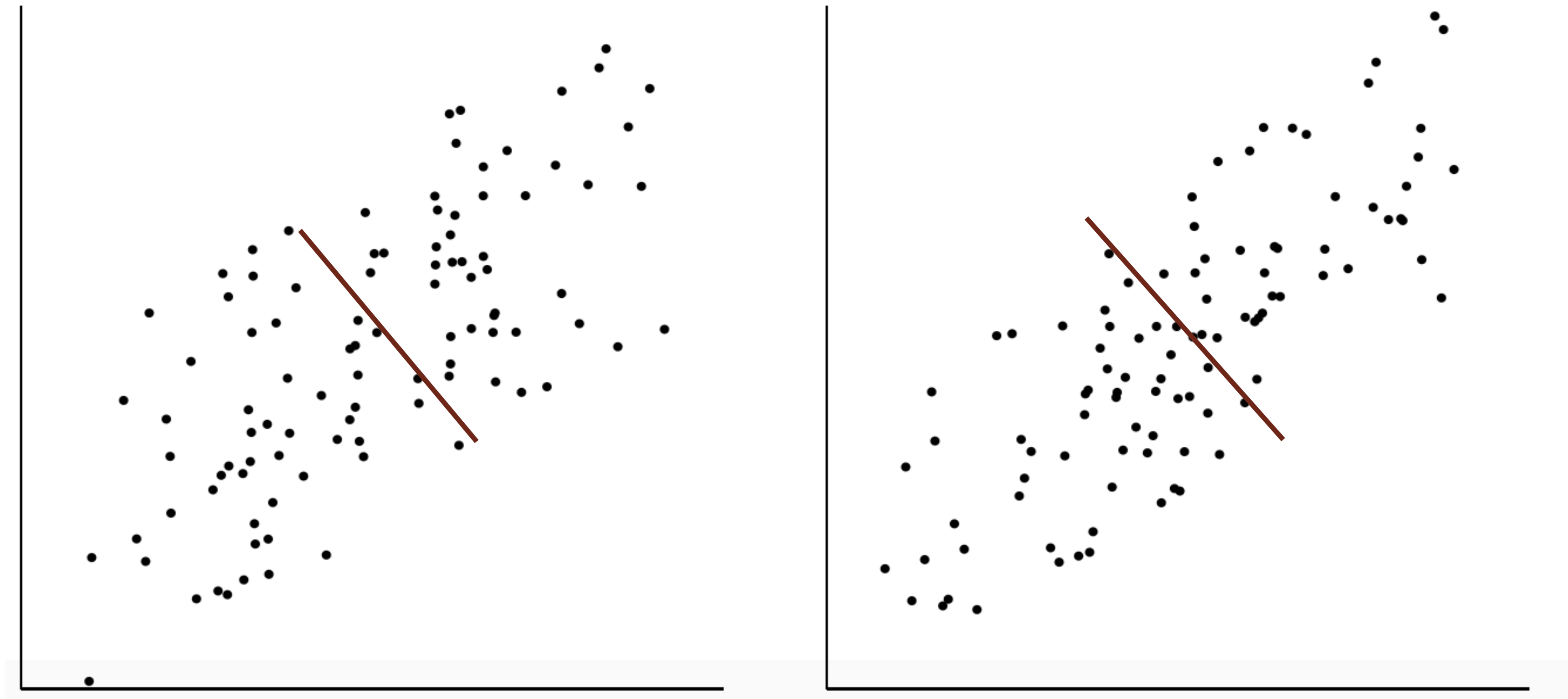
2.2 Hypothesis

- Evidence 3
- Gestalt Psychology
  - Used in user interface design
- The mind forms a global whole with self-organizing tendencies.



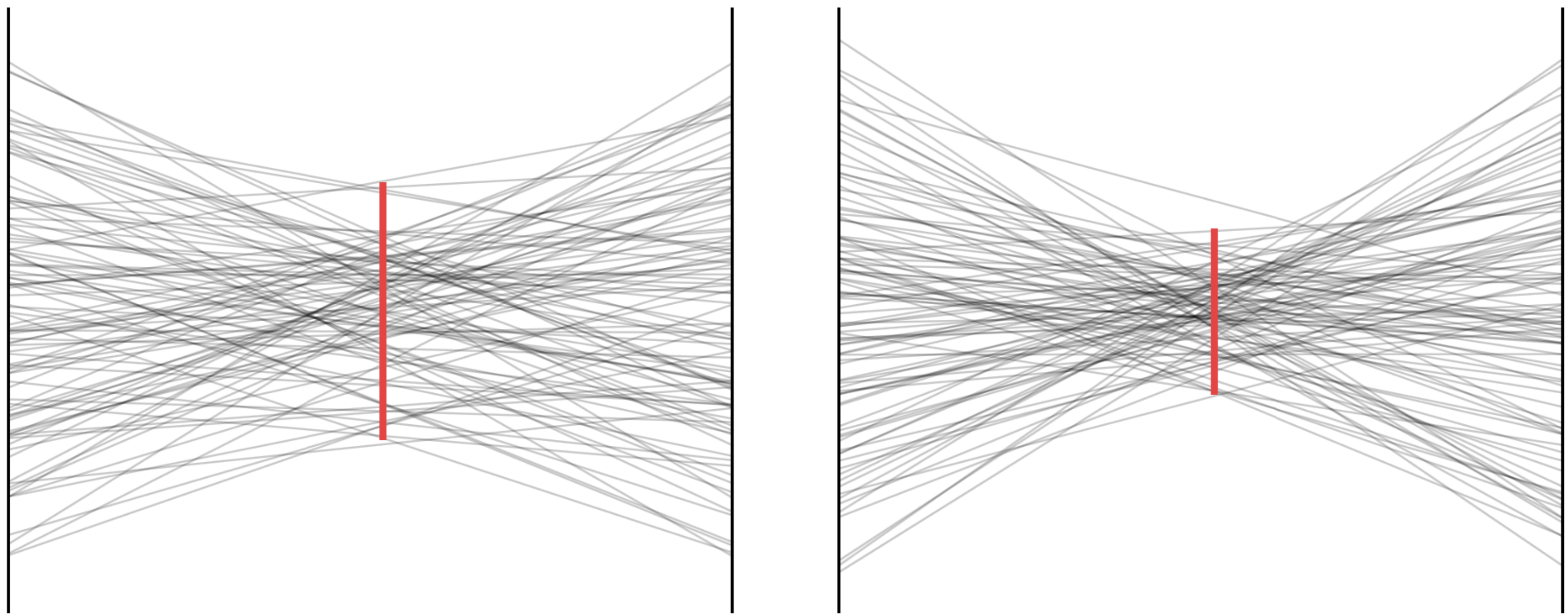
2.2 Hypothesis

Observation



2.2 Hypothesis

Observation



2.2 Hypothesis

- Hypothesis
- A **visual feature** is the substitute of the correlation in judging correlation task

2.2 Hypothesis

- Hypothesis
- A **visual feature** is the substitute of the correlation in judging correlation task
- The perception of correlation follows Weber's law because the perception of the visual feature follows Weber's law

2.2 Hypothesis

- Hypothesis
- A visual feature is the substitute of the correlation in judging correlation task
- Using scatterplots on positive correlated dataset as an example

2 Visual Feature

2.3 Test Hypothesis

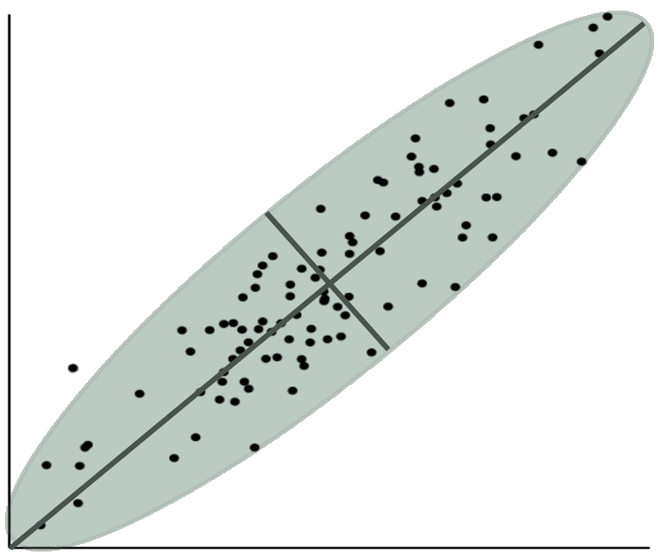
- Collect Visual Features
  - Literature, experts, participants’ comments and brainstorm
  - 81 visual features

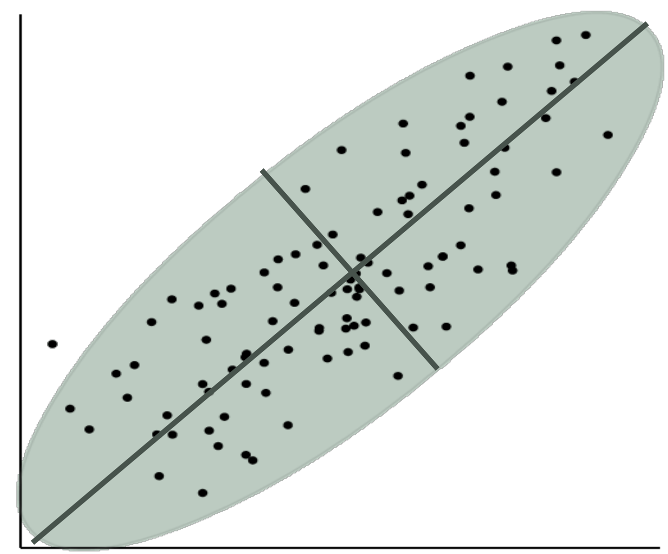
2.3 Test Hypothesis

- Categorize visual features
- 3 categories
  - Length
  - Shape (ratio)
  - Density

2.3 Test Hypothesis

- Length - Axis of the prediction ellipse* **

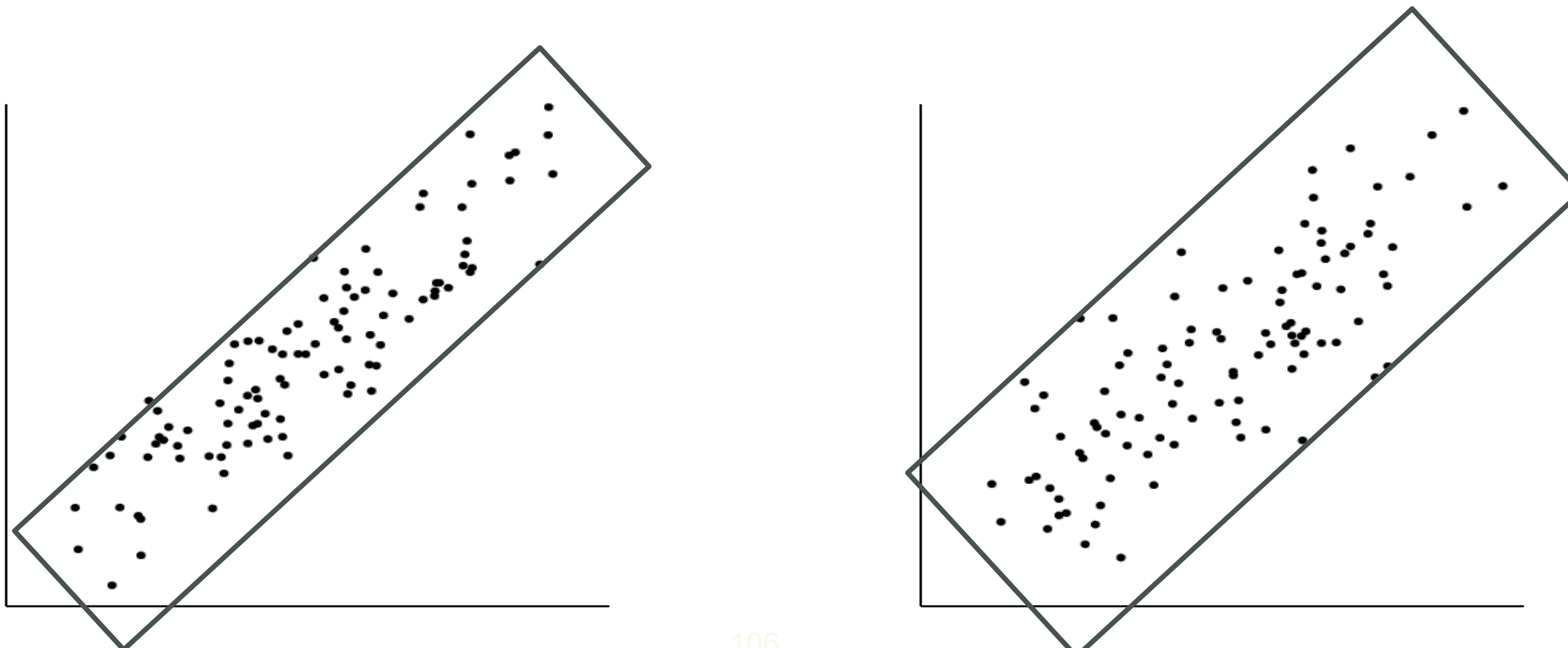




* Alexandersson, Anders. "Graphing confidence ellipses: An update of ellip for Stata 8." Stata Journal 4 (2004): 242-256.
** Rocchi, Marco Bruno Luigi, et al. "The misuse of the confidence ellipse in evaluating statokinesigram." Ital J Sport Sci 12.2 (2005): 169-172

2.3 Test Hypothesis

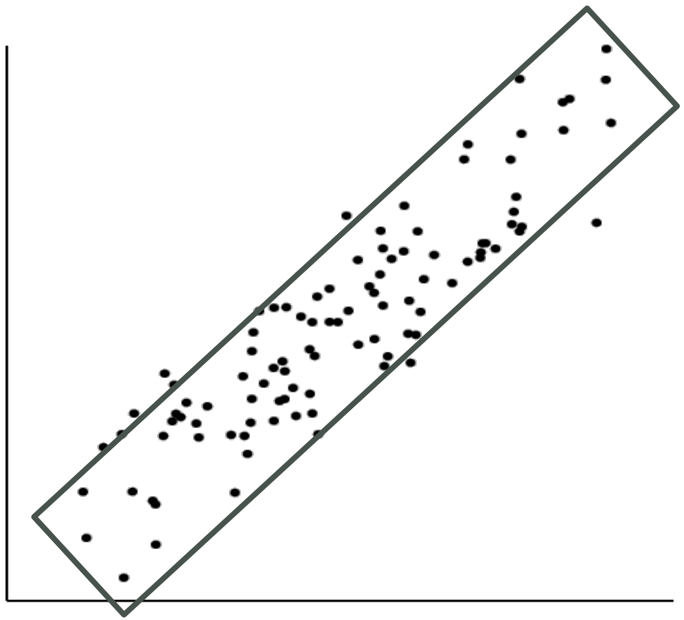
- Length - Sides on the bounding box

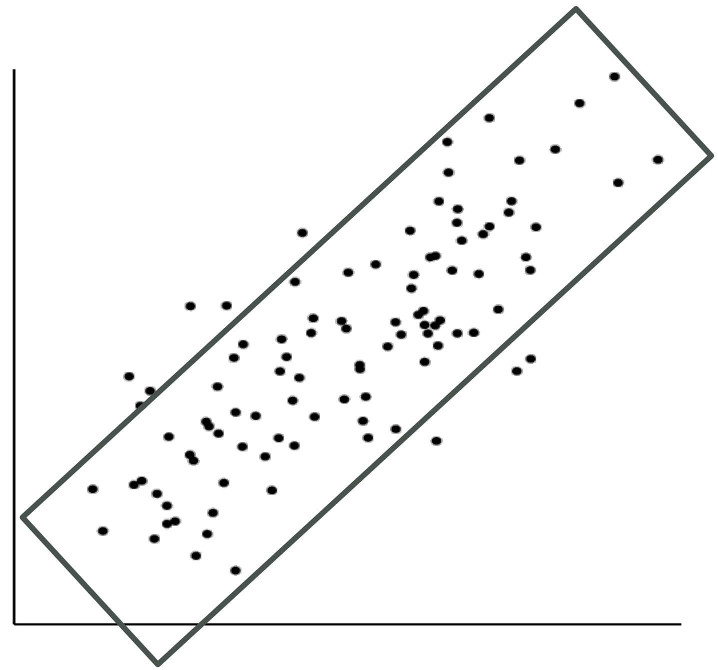


2.3 Test Hypothesis

- Length - Sides on the confidence bounding box

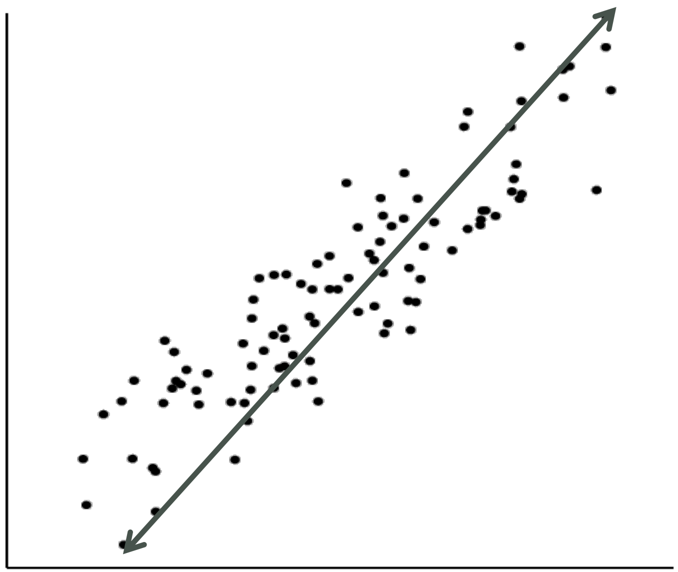
To exclude outliers

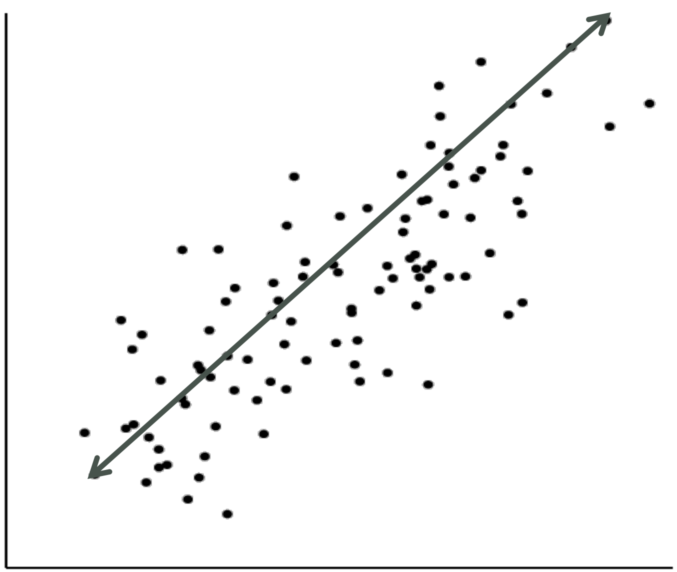




2.3 Test Hypothesis

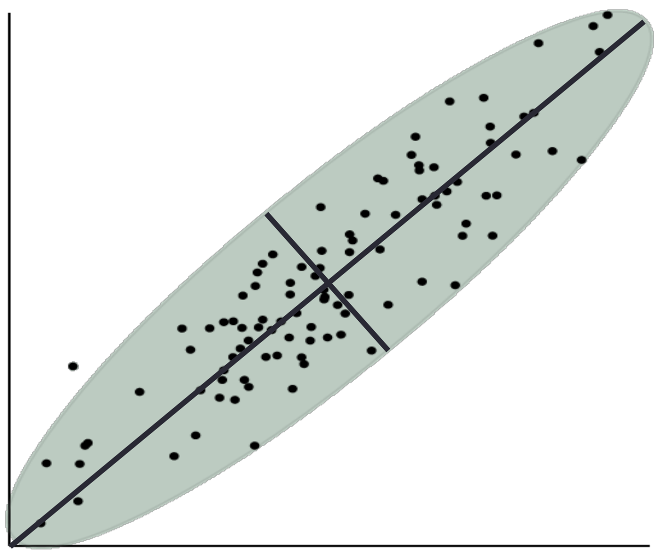
- Length - Max of pairwise distance

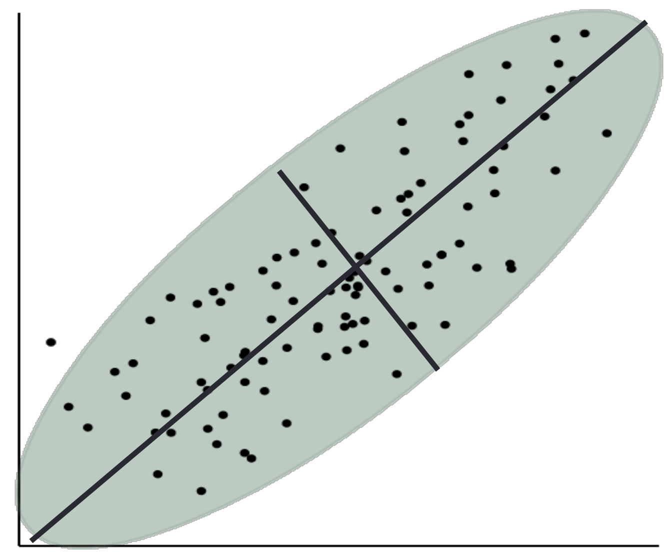




2.3 Test Hypothesis

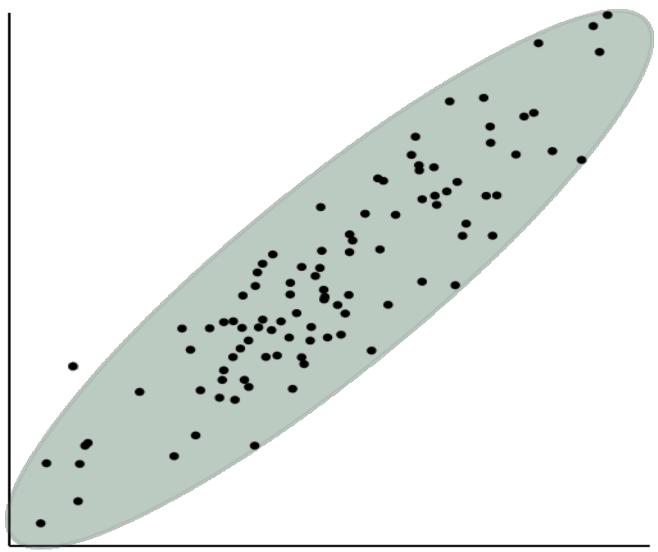
- Shape(ratio) - Ratio of the axes of prediction ellipse

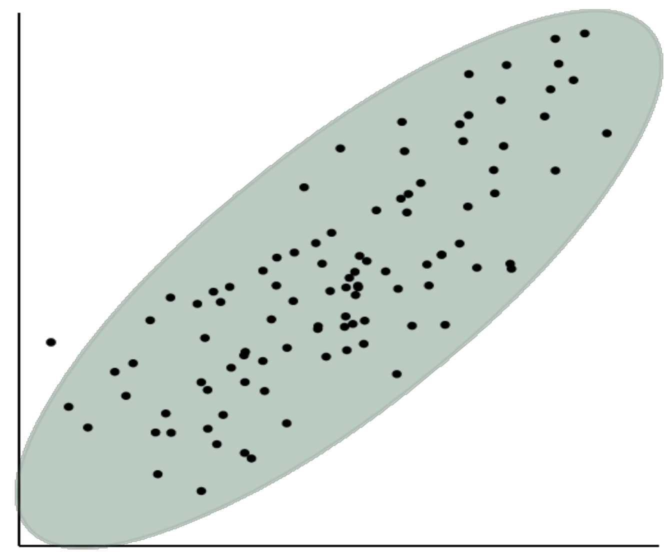




2.3 Test Hypothesis

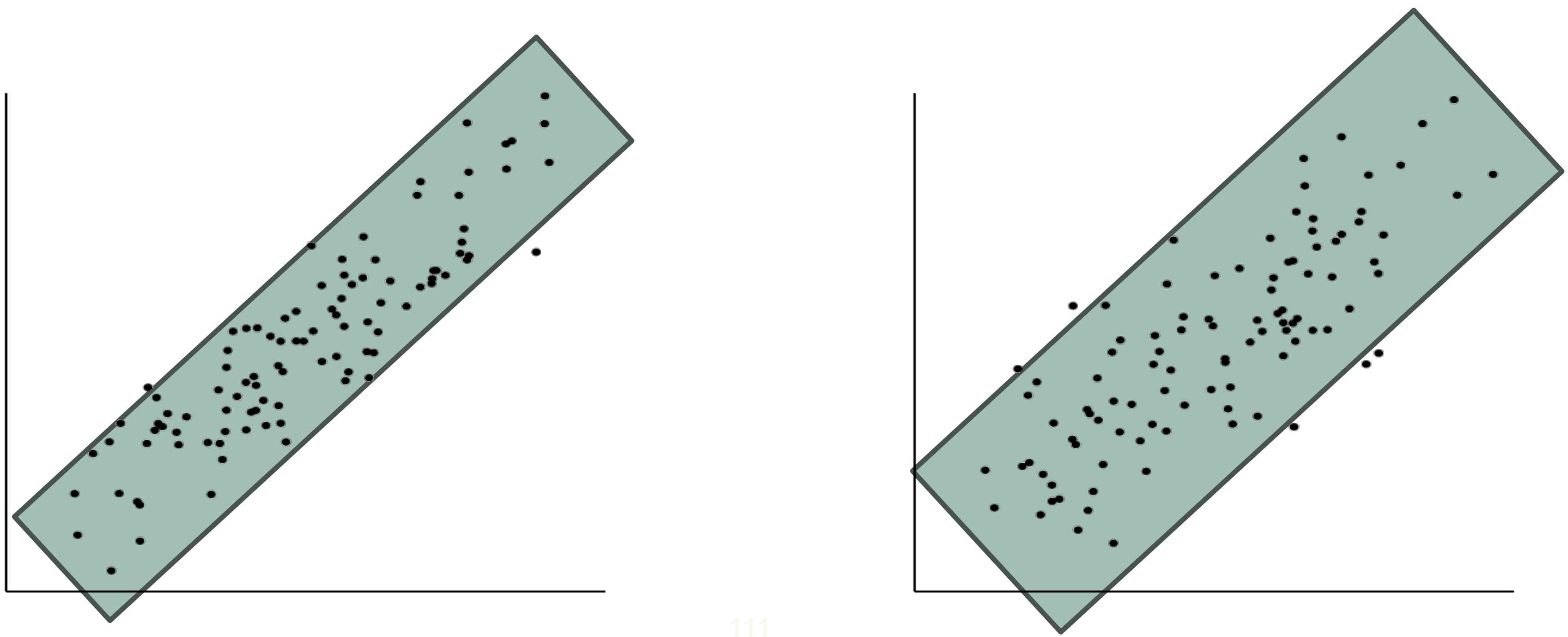
- Shape(ratio) - Area of the prediction ellipse





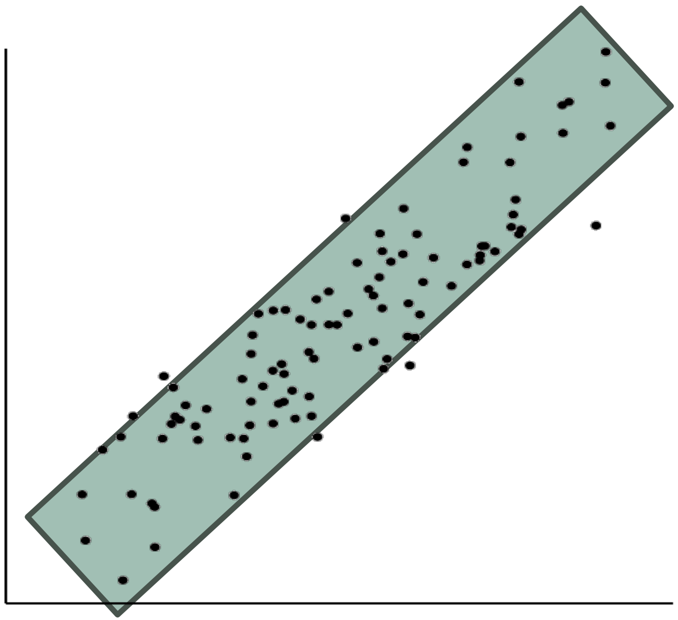
2.3 Test Hypothesis

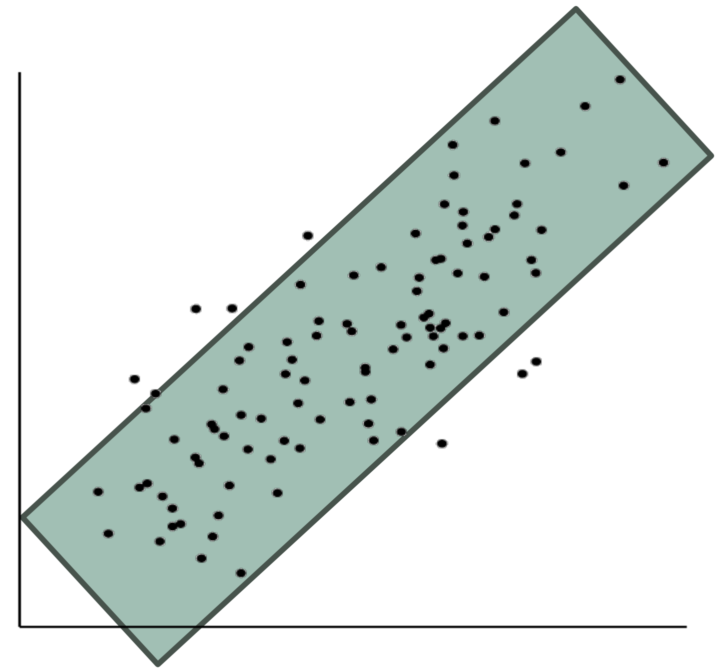
- Shape(ratio) - Area of the bounding box



2.3 Test Hypothesis

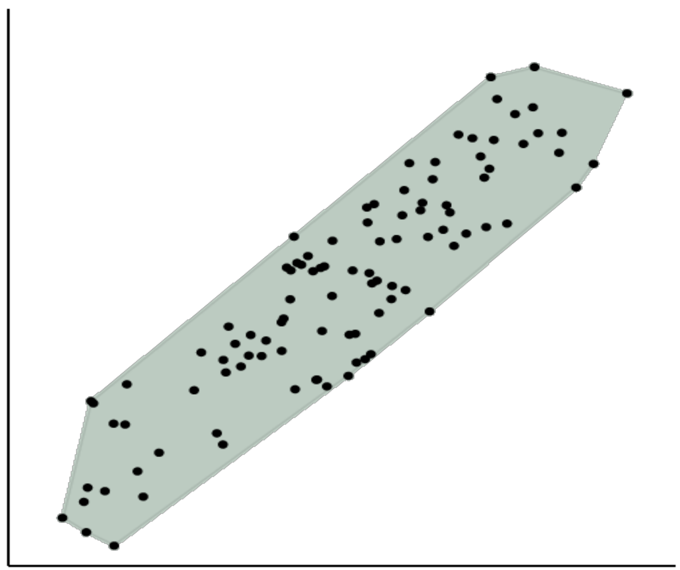
- Shape(ratio) - Area of confidence bounding box

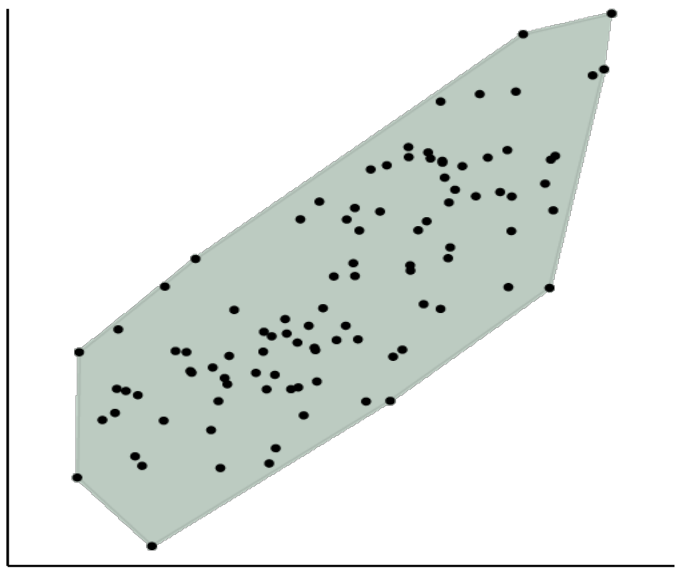




2.3 Test Hypothesis

- Shape(ratio) - Area of convex hull





2.3 Test Hypothesis

Density

- Measurement of skewedness of pairwise distance
- $c = (q_{90} - q_{50}) / (q_{90} - q_{10})$, q is quantile*

* Wilkinson, Leland, Anushka Anand, and Robert L. Grossman. "Graph-Theoretic Scagnostics." INFOVIS. Vol. 5. 2005.

2.3 Test Hypothesis

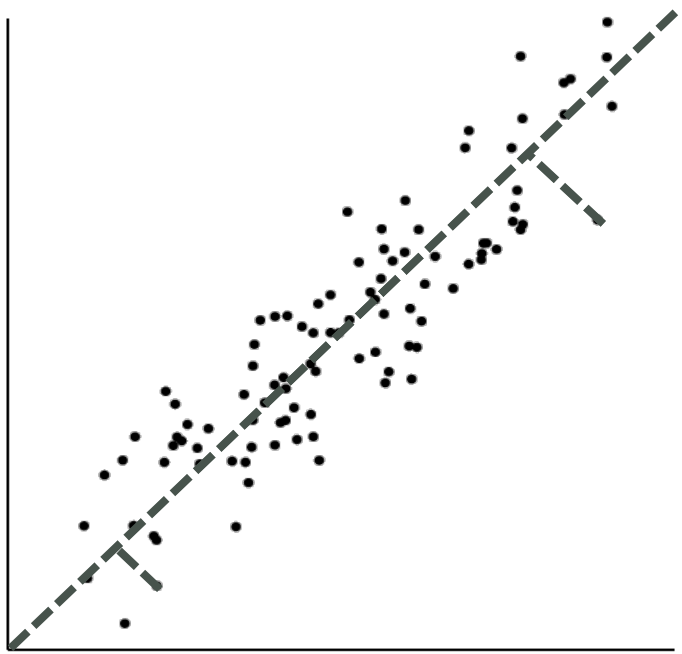
Density

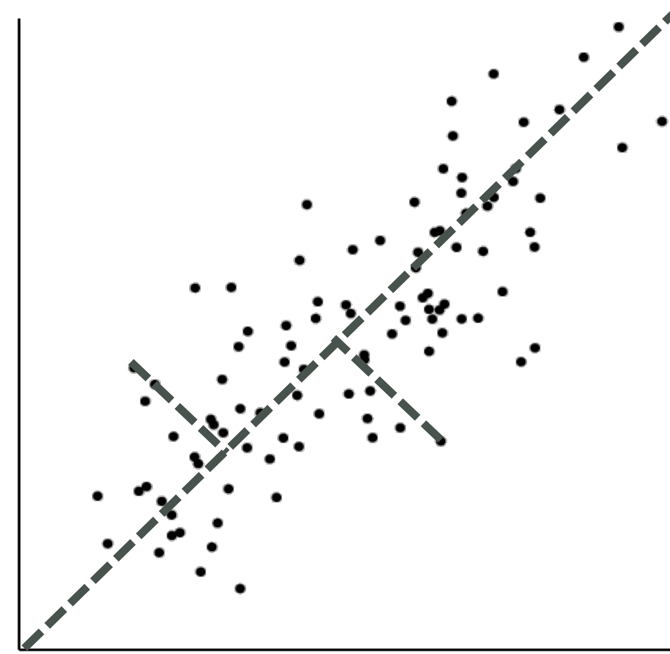
- Measurement of skewedness of pairwise distance on MST
- To exclude outliers
- $c = (q_{90} - q_{50}) / (q_{90} - q_{10})$, q is quantile *

* Wilkinson, Leland, Anushka Anand, and Robert L. Grossman. "Graph-Theoretic Scagnostics." INFOVIS. Vol. 5. 2005.

2.3 Test Hypothesis

- Density - Average distance to the regression line





2.3 Test Hypothesis

- 81 visual features
- Test the hypothesis
  - a visual feature is the substitute of correlation
  - which one is the visual feature

    that the participants are using as the substitute of correlation in judging correlation task?

2.3 Test Hypothesis

6 Criteria

- Experimental data
- Model fit result

2.3 Test Hypothesis

Experimental Data

Criterion 1 : The difference of the visual feature predicts the participants' judgment

Criterion 2 : The convergence of the visual feature is consistent with the convergence of r

Model Fits

Criterion 3 : The magnitude of the visual feature is correlated with the magnitude of correlation r

Criterion 4 : The discrimination threshold of the visual feature is consistent the discrimination threshold of correlation

Criterion 5 : The visual feature follows the Weber's law

Criterion 6 : The Weber equation of the visual feature is consistent with the Weber equation of r

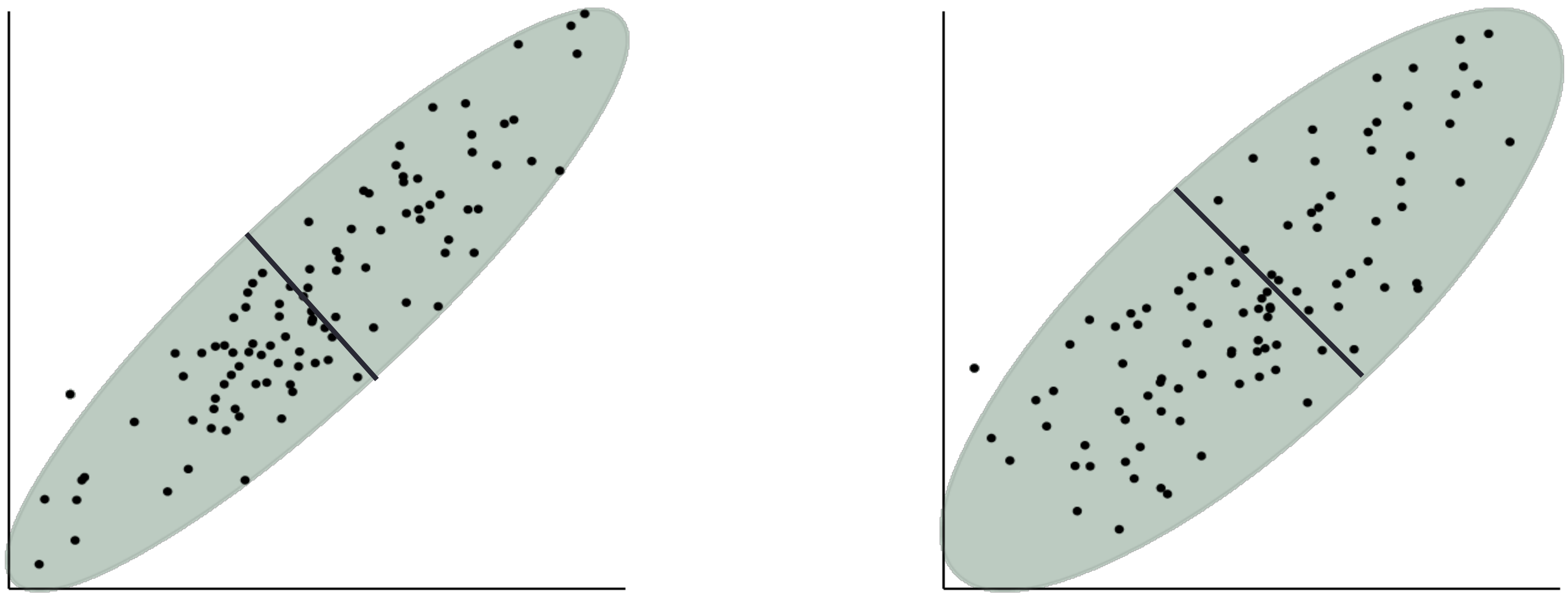
2.3 Test Hypothesis

- 6 criteria → 2 as example
  - Criterion 1 & 6
- 81 visual features → 2 as example
  - Minor axis of prediction ellipse
  - Convex hull

- How to judge if a visual feature is the one that the participants are using in judgment correlation task using the criteria

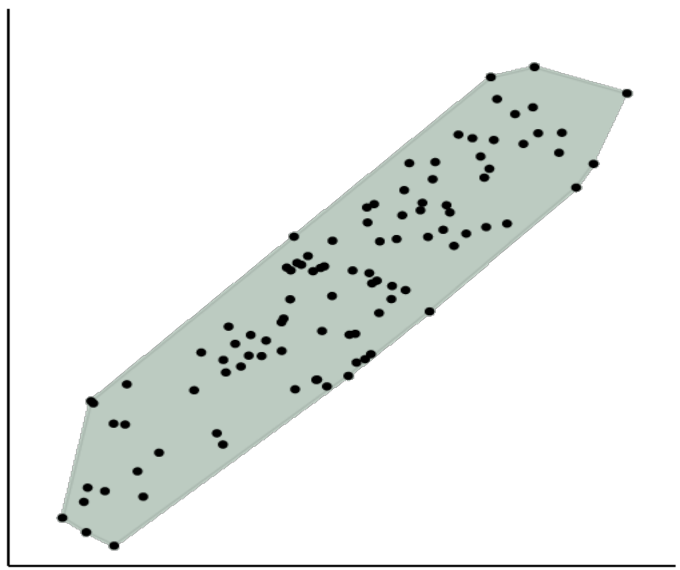
2.3 Test Hypothesis

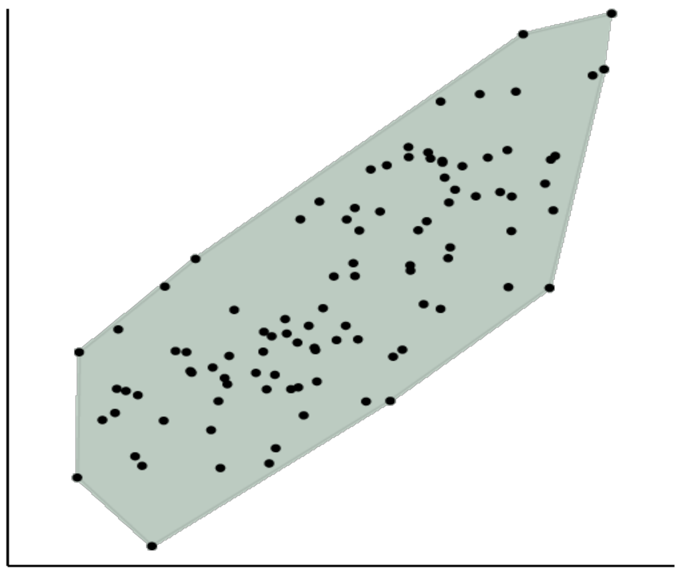
- Example of visual features
- The minor axis of prediction ellipse



2.3 Test Hypothesis

- Example of visual features
- Shape(ratio) - Area of convex hull



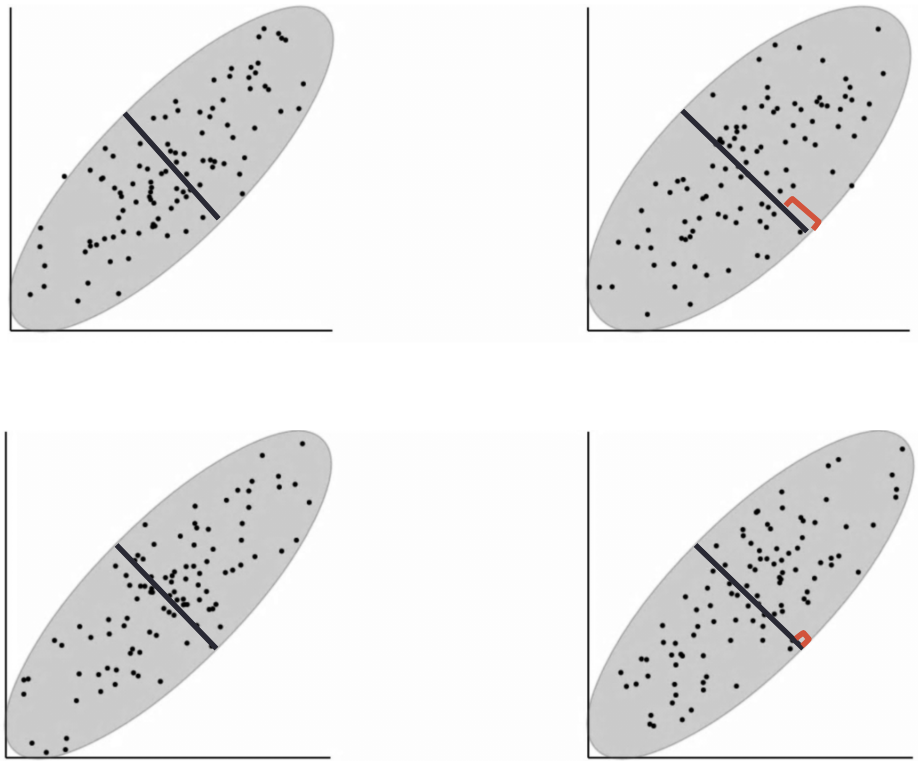


2.3 Test Hypothesis

- Criterion 1 (C1)
  - The difference of the visual feature predicts the participants' judgment

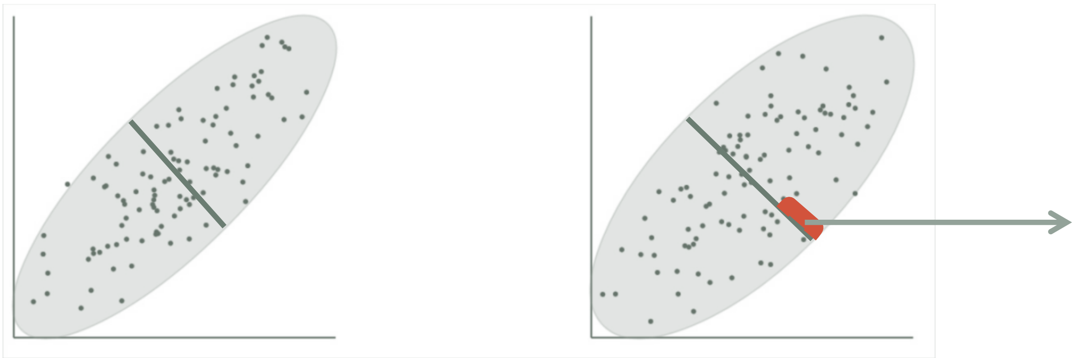
2.3 Test Hypothesis

- The difference of the visual feature predicts the participants' judgment



2.3 Test Hypothesis

- The difference of the visual feature predicts the participants' judgment

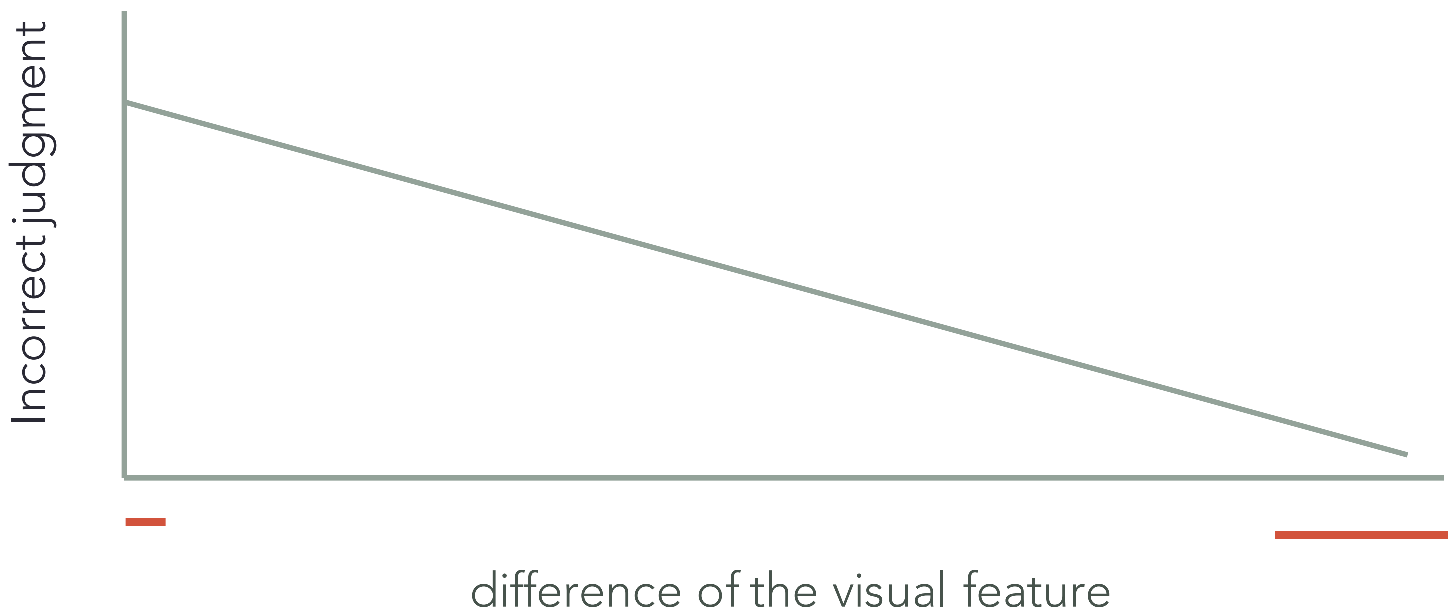


Easy to tell
High frequency of correct judgment

Hard to tell
Low frequency of correct judgment

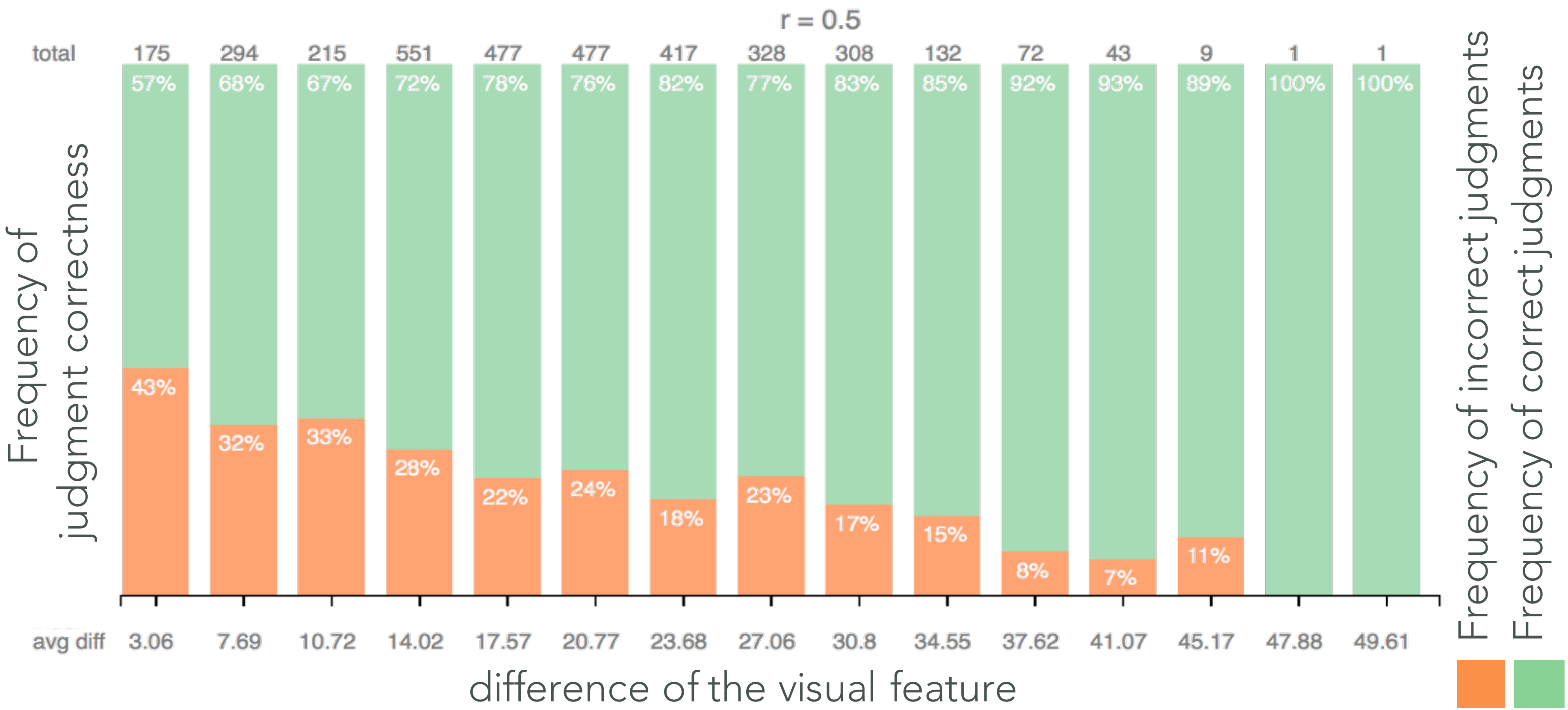
2.2 Test Hypothesis

- Correlation between the difference of the visual feature and judgment correctness



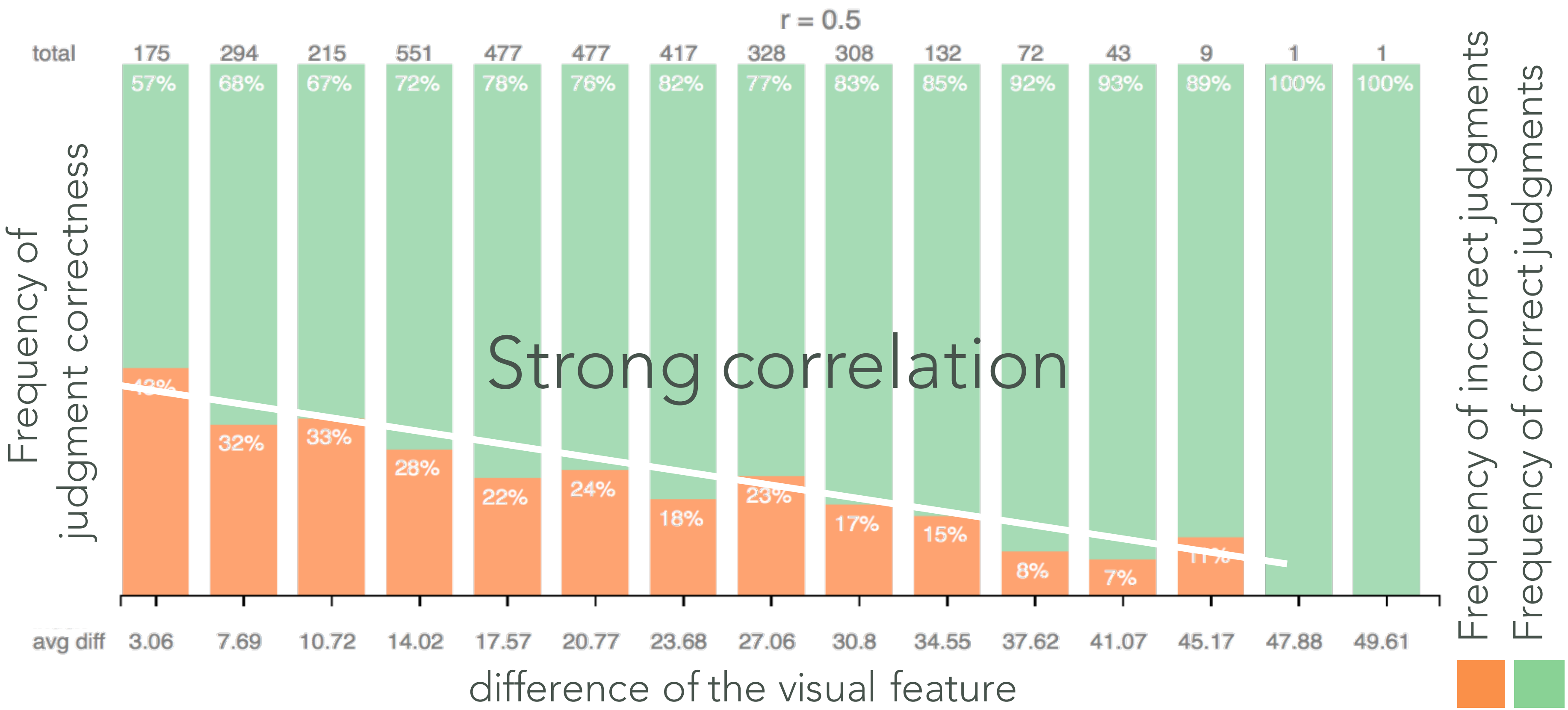
2.3 Test Hypothesis

- Minor axis of prediction ellipse



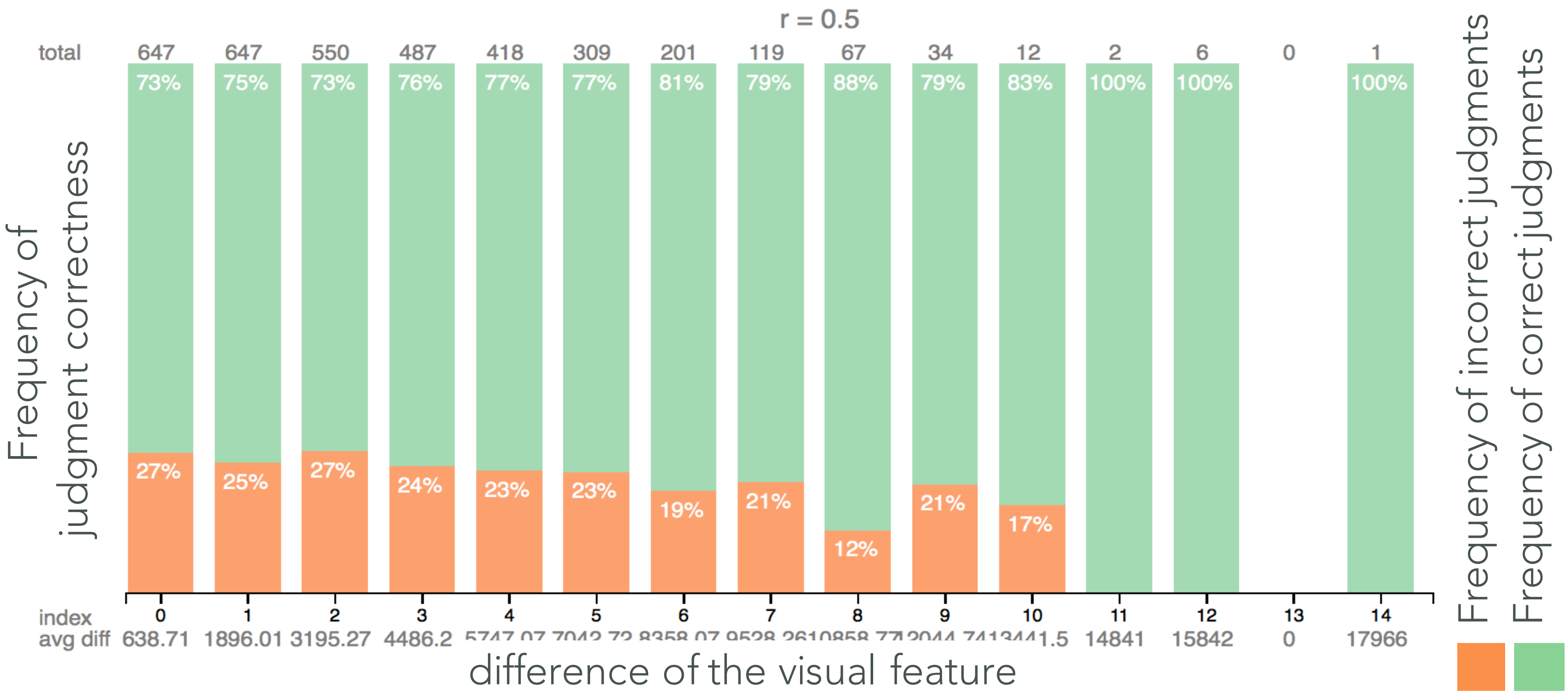
2.3 Test Hypothesis

- Minor axis of prediction ellipse



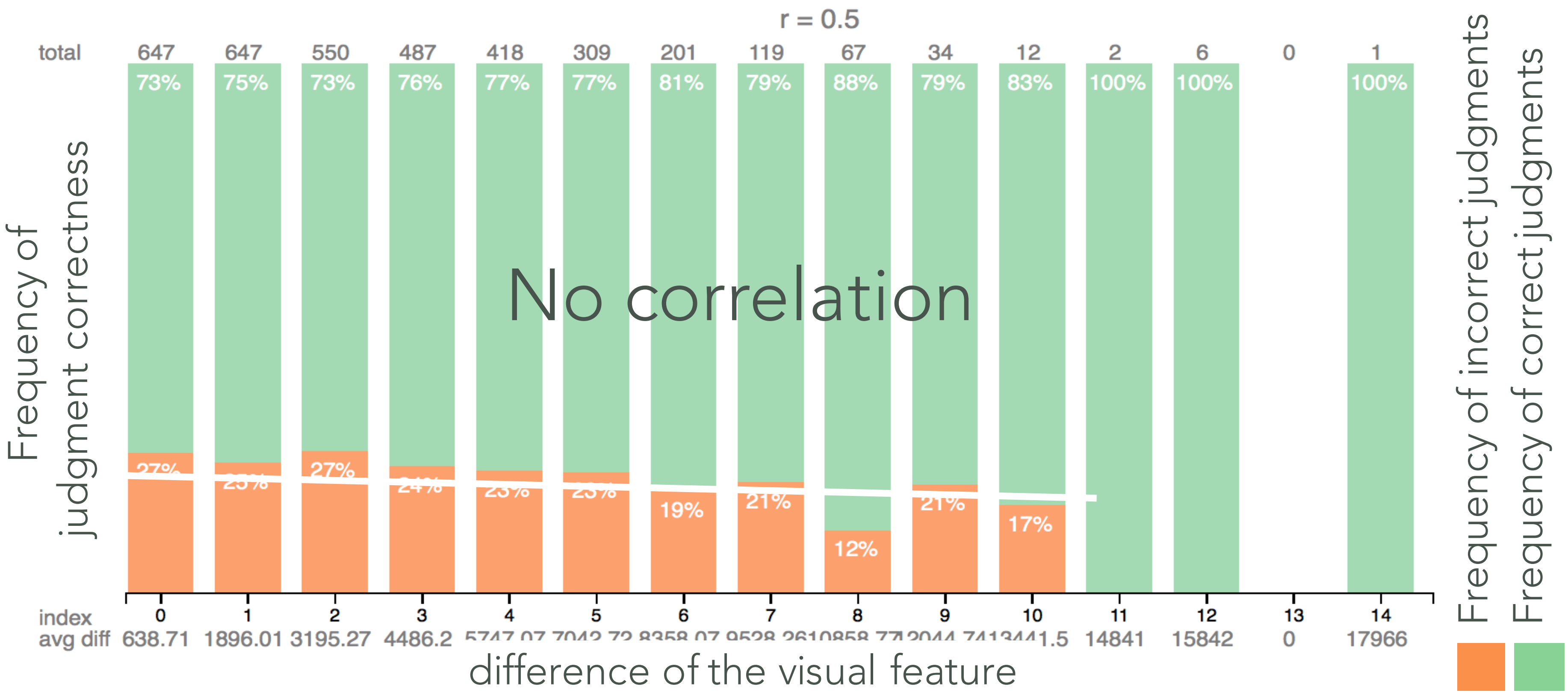
2.3 Test Hypothesis

- Area of convex hull



2.3 Test Hypothesis

- Area of convex hull

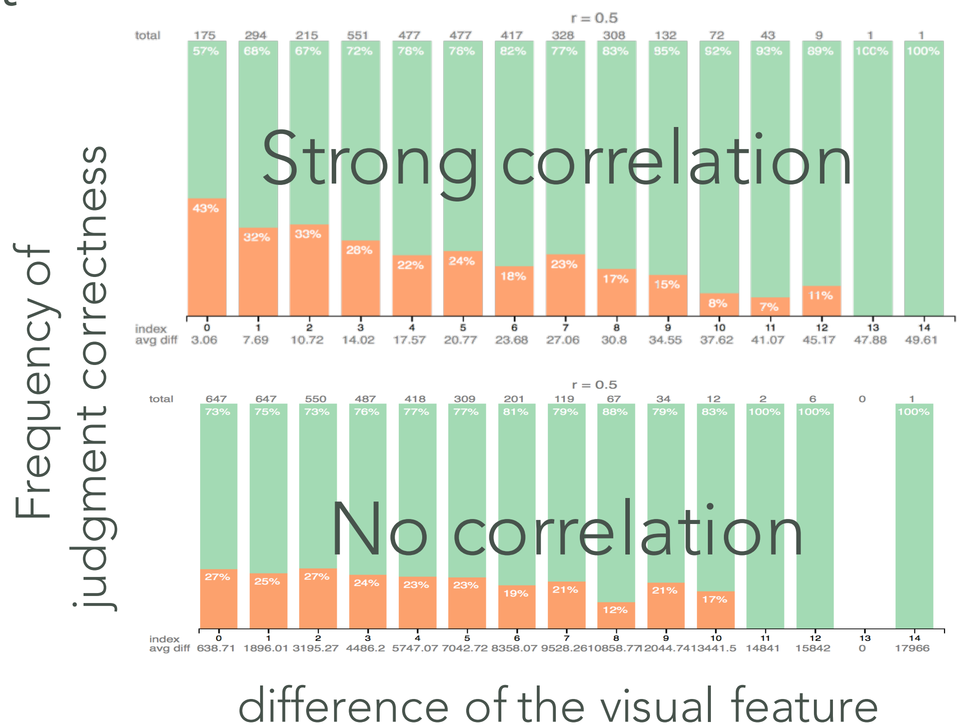


2.3 Test Hypothesis

- C1 The difference of the visual feature predicts the participants' judgment

Minor axis of prediction ellipse

Area of convex hull



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2.3 Test Hypothesis

- Criterion 6 (C6)
  - The Weber equation of the visual feature is consistent with the Weber equation of r

2.3 Test Hypothesis

- Two Weber models
- $JND_v = k_v\, v + b_v$ (visual feature)
- $JND_r = k_r\, r + b_r$ (correlation)

2.3 Test Hypothesis

- Two Weber models
- $JND_v = k_v \, v + b_v$
- $JND_r = k_r \, r + b_r$

If the visual feature is the substitute

2.3 Test Hypothesis

- Two Weber models
- $JND_v = k_v\, v + b_v$
- $JND_r = k_r\, r + b_r$

If the visual feature is the substitute

Substitute the r with the visual feature in Weber model of correlation

2.3 Test Hypothesis

- Two Weber models
- $JND_v = k_v\, v + b_v$
- $JND_r = k_r\, r + b_r$

$JND_v = f(JND_r)$

$v = g(r)$

Computed in previous criteria

Substitute the r with the visual feature in Weber model

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- minor axis of prediction ellipse

  (from previous criteria)

$$\boxed{JND_v = 0.9959\ JND_r + 7.3666}$$

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- minor axis of prediction ellipse

(from previous criteria)

$$\boxed{JND_v = 0.9959\ JND_r + 7.3666}$$

(from previous criteria)

$$\boxed{v = -207.5829\ r + 310.8461}$$

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- minor axis of prediction ellipse

(from previous criteria)

$$\boxed{JND_v = 0.9959\ JND_r + 7.3666}$$

(from previous criteria)

$$\boxed{v = -207.5829\ r + 310.8461}$$

(from previous criteria)

$$\boxed{JND_v = 0.1575\ v + -13.3688}$$

(Weber model of the visual feature)

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- minor axis of prediction ellipse

(from previous criteria)

$$\boxed{JND_v = 0.9959\ JND_r + 7.3666}$$

(from previous criteria)

$$\boxed{v = -207.5829\ r + 310.8461}$$

(from previous criteria)

$$\boxed{JND_v = 0.1575\ v + -13.3688}$$

(Weber model of the visual feature)

$$\boxed{JND_r = -0.2543\ r + 0.2196}$$

(Inferred Weber model of the r)

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- minor axis of prediction ellipse

(from previous criteria)

$$JND_v = 0.9959\ JND_r + 7.3666$$

(from previous criteria)

$$v = -207.5829\ r + 310.8461$$

(from previous criteria)

$$JND_v = 0.1575\ v + -13.3688$$

(Weber model of the visual feature)

$$JND_r = -0.2543\ r + 0.2196$$

(Inferred Weber model of the r)

$$JND_r = -0.2713\ r + 0.2325$$

(Original Weber model of the r)

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- minor axis of prediction ellipse

(from previous criteria)

$$JND_v = 0.9959\ JND_r + 7.3666$$

(from previous criteria)

$$v = -207.5829\ r + 310.8461$$

(from previous criteria)

$$JND_v = 0.1575\ v + -13.3688$$

(Weber model of the visual feature)

$$JND_r = -0.2543\ r + 0.2196$$

(Inferred Weber model of the r)

$$JND_r = -0.2713\ r + 0.2325$$

(Original Weber model of the r)

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- Area of convex hull

  (from previous criteria)

$$\boxed{JND_v = 1708.9382\ JND_r + 4175.9671}$$

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- Area of convex hull

(from previous criteria)

$$\boxed{JND_v = 1708.9382\ JND_r + 4175.9671}$$

(from previous criteria)

$$\boxed{v = -28161.2269\ r + 56355.0590}$$

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- Area of convex hull

(from previous criteria)

$$\boxed{JND_v = 1708.9382\ JND_r + 4175.9671}$$

(from previous criteria)

$$\boxed{v = -28161.2269\ r + 56355.0590}$$

(from previous criteria)

$$\boxed{JND_v = 0.02275\ v + 3419.6650}$$

(Weber model of the visual feature)

2.3 Test Hypothesis

- (C6) Substitute the r with the visual feature in Weber model
- Area of convex hull

(from previous criteria)

$$JND_v = 1708.9382 \ JND_r + 4175.9671$$

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(Weber model of the visual feature)

$$JND_r = -0.3749 \ r + 0.3077$$

(Inferred Weber model of the r)

2.3 Test Hypothesis

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(Weber model of the visual feature)

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(Inferred Weber model of the r)

$$JND_r = -0.2713\ r + 0.2325$$

(Original Weber model of the r)

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- (C6) Substitute the r with the visual feature in Weber model
- Area of convex hull

(from previous criteria)

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$$JND_v = 0.02275\ v + 3419.6650$$

(Weber model of the visual feature)

$$JND_r = -0.3749\ r + 0.3077$$

(Inferred Weber model of the r)

$$JND_r = -0.2713\ r + 0.2325$$

(Original Weber model of the r)

2.3 Test Hypothesis

- (C6) The Weber equation of the visual feature is consistent with the Weber equation of r

Minor axis of prediction ellipse

$$JND_r = -0.2543\ r + 0.2196$$

(Inferred weber model of the r)

√

Area of convex hull

$$JND_r = -0.3749\ r + 0.3077$$

(Inferred weber model of the r)

×

$$JND_r = -0.2713\ r + 0.2325$$

(Original weber model of the r)

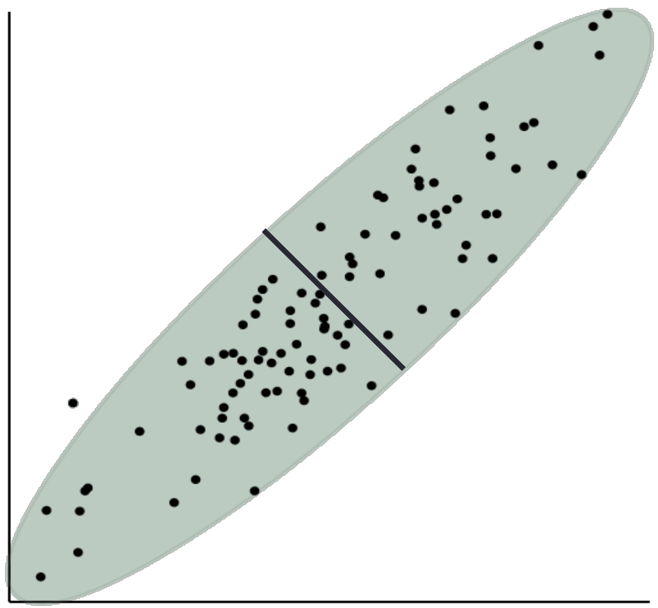
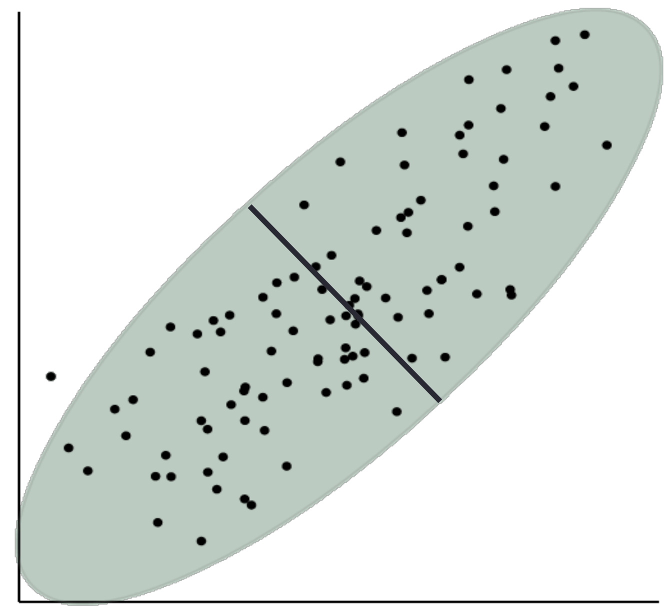
2.3 Test Hypothesis

- 81 Visual Features
- 6 criteria

- 5 passed all but with different scores
- Each of the other 76 failed in at least one criterion
- Not the visual feature

2.3 Test Hypothesis

- Currently, the BEST one is
- Minor axis of the prediction ellipse

2.3 Test Hypothesis

- Hypothesis
- A **visual feature** is the substitute of the correlation in judging correlation task

2.3 Test Hypothesis

- Hypothesis
- A **visual feature** is the substitute of the correlation in judging correlation task
- Use **scatterplots** for **positive correlated dataset** as an example

2.3 Test Hypothesis

- Hypothesis
- A **visual feature** is the substitute of the correlation in judging correlation task
- → QED

2.3 Test Hypothesis

- Hypothesis
- A **visual feature** is the substitute of the correlation in judging correlation task
- Minor axis of the prediction ellipse

2 Visual Feature

- 2.1 Context
- 2.1 Contribution
- 2.2 Hypothesis
- 2.3 Test Hypothesis
- 2.4 Implication
- 2.5 Summary

2.4 Implication

- Contribution

2.4 Implication

- 81 visual features
- 6 criteria
- To…

2.4 Implication

- Why does the Weber’s law work for correlation?

2.4 Implication

- Why does the Weber's law work for correlation?
  - A visual feature is used as a substitute of correlation in judging correlation task

2.4 Implication

- Why does the Weber's law work for correlation?
  - A visual feature is used as a substitute of correlation in judging correlation task
  - Scatterplots
  - Minor axis of the prediction ellipse

2.4 Implication

- Why does the Weber's law work for correlation?
  - Perceiving the minor axis of the ellipse
  - Correlation → Length

2.4 Implication

- Why does the Weber's law work for correlation?
  - Perceiving the minor axis of the ellipse
  - Correlation → Length
  - Length → Weber's Law

2.4 Implication

- Why does the Weber's law work for correlation?
  - Perceiving the minor axis of the ellipse
  - Correlation → Length
  - Length → Weber's Law
  - Correlation → Weber's Law

2.4 Implication

- To generalize the work?
  - Larger canvas, more data points, larger point size
  - The minor axis is recognizable
  - Weber's law holds

  - Future Work: To validate the effect when varying the parameters, i.e. canvas size

2.4 Implication

- To generalize the work?
  - This is a semantic work to measure if a visual feature is the substitute of a measurement

2.4 Implication

- Why does a visualization work?
  - In the case of scatterplots to show correlation
  - Scatterplots communicate with people using the visual feature
  - Minor axis of the prediction ellipse

2 Visual Feature

- 2.1 Contribution
- 2.2 Hypothesis
- 2.3 Test Hypothesis
- 2.4 Implication
- 2.5 Summary

2.5 Summary

- Why does the Weber's law work for correlation?
  - A visual feature is used as the substitute of correlation in judging correlation task
  - 81 visual features
  - 6 criteria

Overview

1. Perceptual Model of Visualization
   - Quantify visual limitations using perception law
2. Visual Feature (On-going)
   - Generalize perceptual models of visualization
3. Visual-Centric Computation (Future Work)
   - Use visual limitations to guide computation

3 Visual-Centric Computation

- 3.1 What's Visual-Centric Computation
- 3.2 Why Visual-Centric Computation
- 3.3 How to do Visual-Centric Computation

3.1 What's Visual-Centric Computation

- Use visual limitations to guide computation

3.1 What's Visual-Centric Computation

- Use ~~visual limitations~~ to guide computation

Quantified as

perceptual models

3.1 What's Visual-Centric Computation

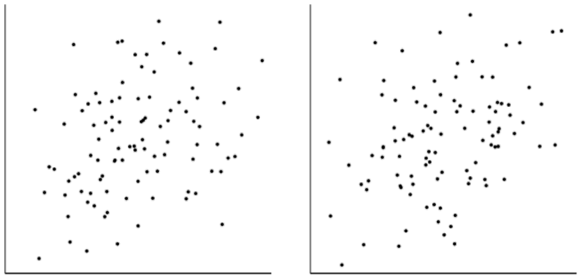
- Use perceptual models to guide computation

3.1 What's Visual-Centric Computation

- Perceptual models
  - Lexical level
    - i.e. pixel level JND

Uncompressed JPG2000

  - Semantic level
    - statistical measurements
    - i.e. JND of correlation



3 Visual-Centric Computation

- 3.1 What's Visual-Centric Computation
- 3.2 Why Visual-Centric Computation
- 3.3 How to do Visual-Centric Computation


Luc Legay
Label your
Professional Networks
Social media
Medias, comm, contenus
Divers
Explorateurs éclaireurs
Les historiques
Fing, institutions, recherch
Experts & consultants
Marketing, eMarketing

3.2 Why Visual-Centric Computation

- Computation on big data
  - Expensive in time & resource
  - Visualization of the result → not understandable

3.2 Why Visual-Centric Computation

Traditional Computation

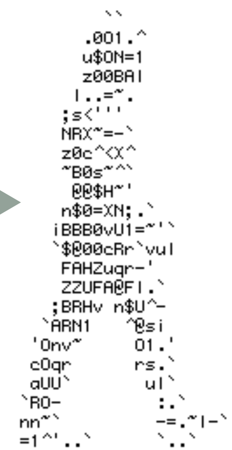
Computation

Result

3.2 Why Visual-Centric Computation

Traditional Computation

Result



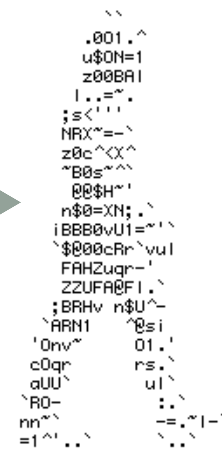
Computation

Render
visualization

3.2 Why Visual-Centric Computation

Traditional Computation

Result



Image

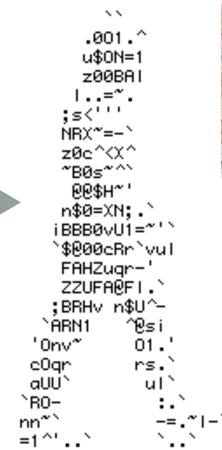
Computation

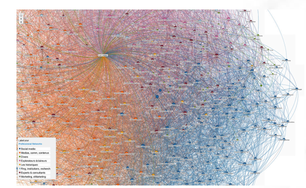
Render visualization

3.2 Why Visual-Centric Computation

Traditional Computation

Result





?!?

Computation

Render visualization

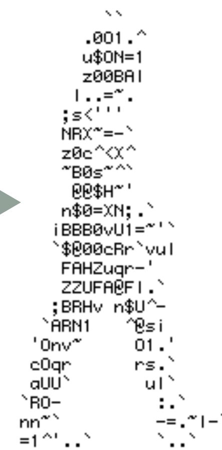
Limitations in human and screen resolution

3.2 Why Visual-Centric Computation

Visual-Centric Computation

Computation

Result →



Render visualization

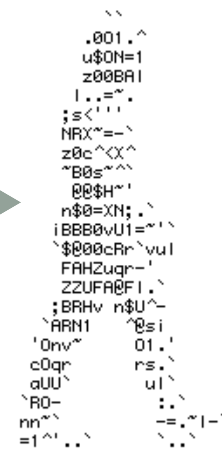
Image →

Limitations in human and screen resolution

3.2 Why Visual-Centric Computation

Visual-Centric Computation

Result



Image

Computation

Render visualization

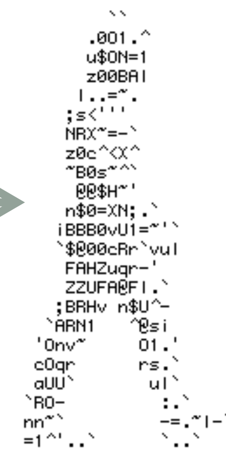
Limitations in human and screen resolution

3.2 Why Visual-Centric Computation

Visual-Centric Computation

Computation

Result

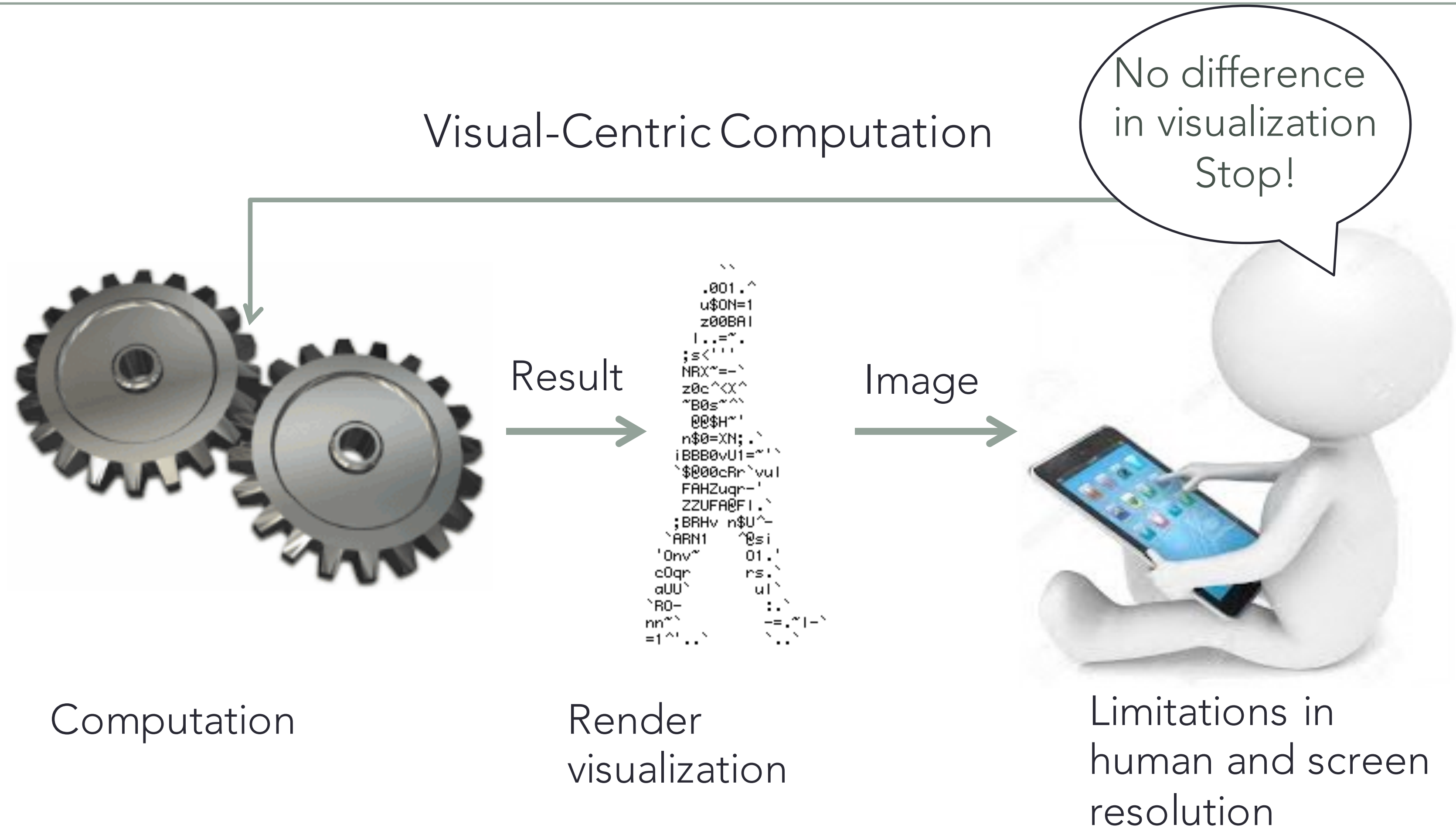


Render visualization

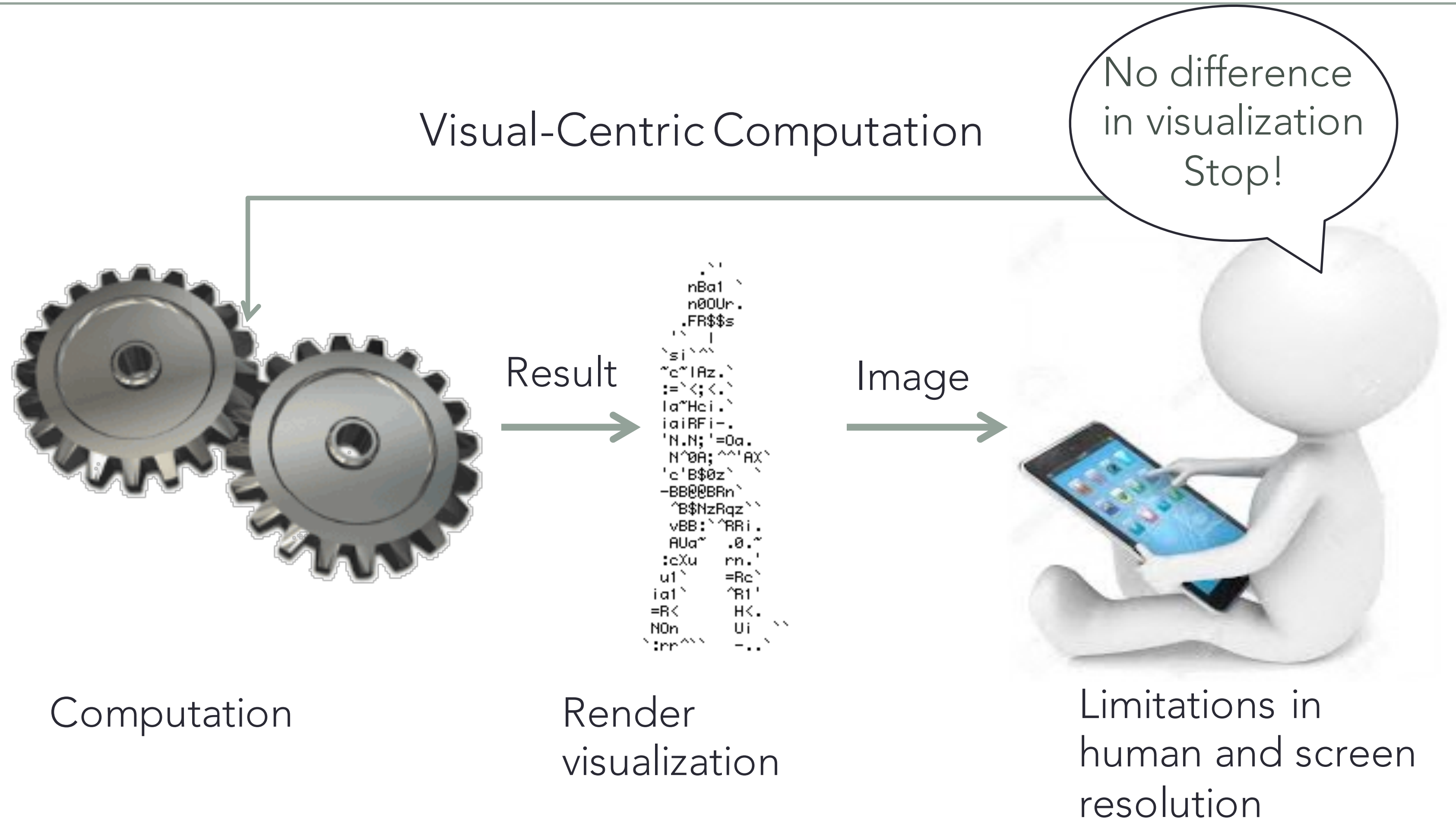
Image

Limitations in human and screen resolution

3.2 Why Visual-Centric Computation



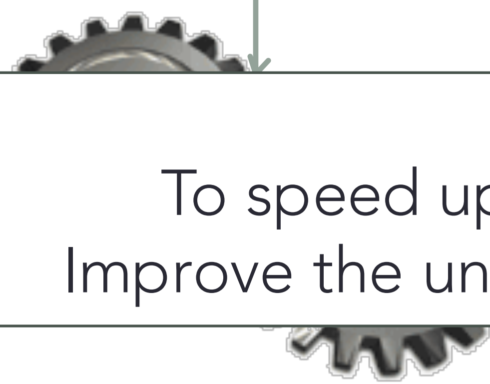
3.2 Why Visual-Centric Computation

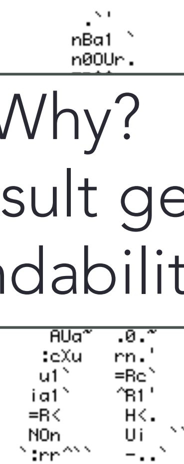


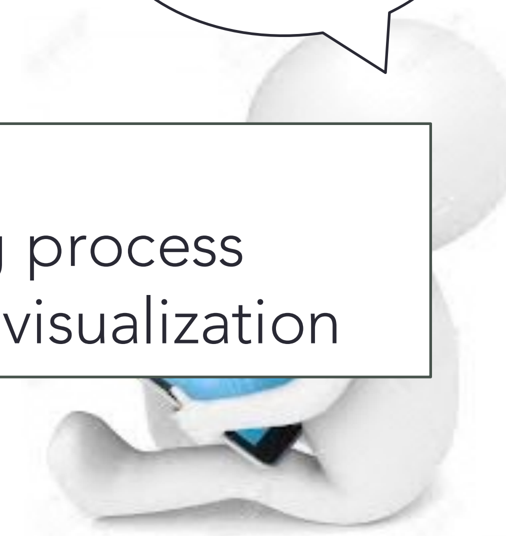
3.2 Why Visual-Centric Computation

Visual-Centric Computation

No difference
in visualization
Stop!







Why?
To speed up the result generating process
Improve the understandability of the visualization

Computation

Render
visualization

Limitations in
human and screen
resolution

3 Visual-Centric Computation

3.3 How to do Visual-Centric Computation

- Perceptual Model based
- Sampling
- Approximate computation

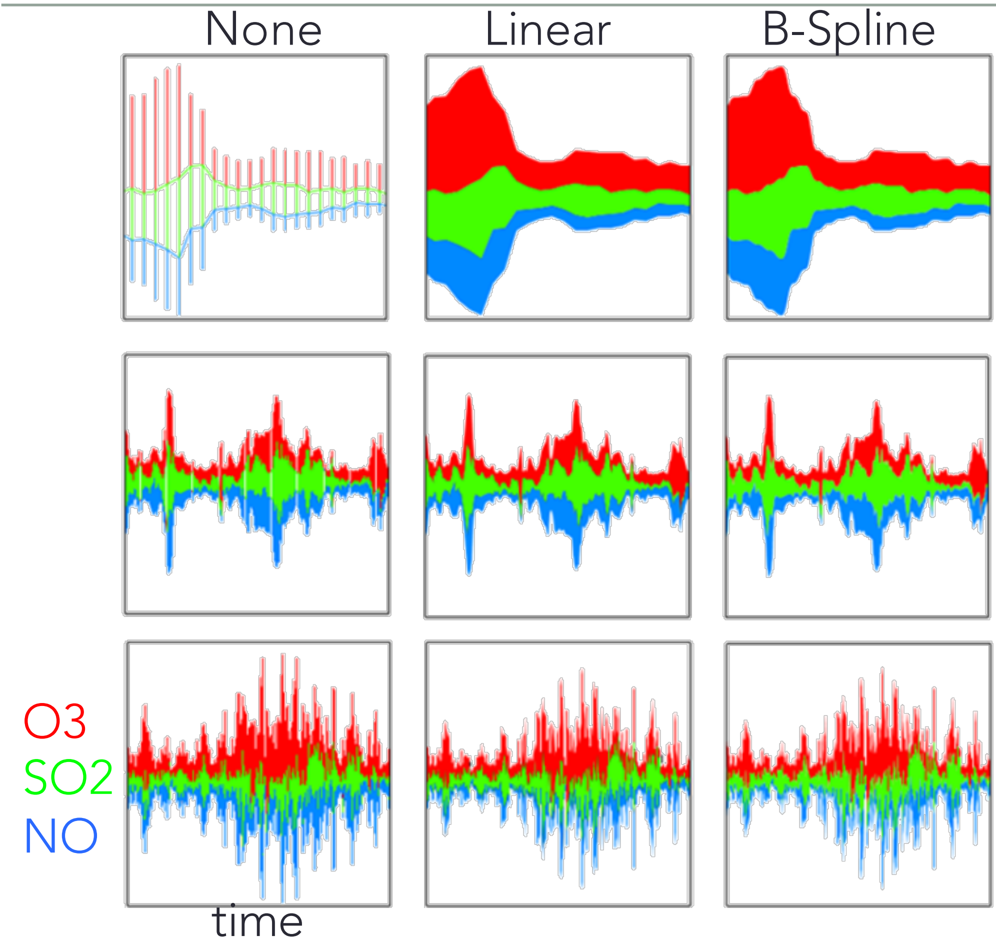
3.3 How to do Visual-Centric Computation

- Sampling
  - Until human can’t see the lexical level difference
    - pixel level JND
    - JPEG – compress until can’t tell pixel level difference
  - Until human can’t see the semantic difference
    - semantic level JND
    - correlation

3.3 How to do Visual-Centric Computation

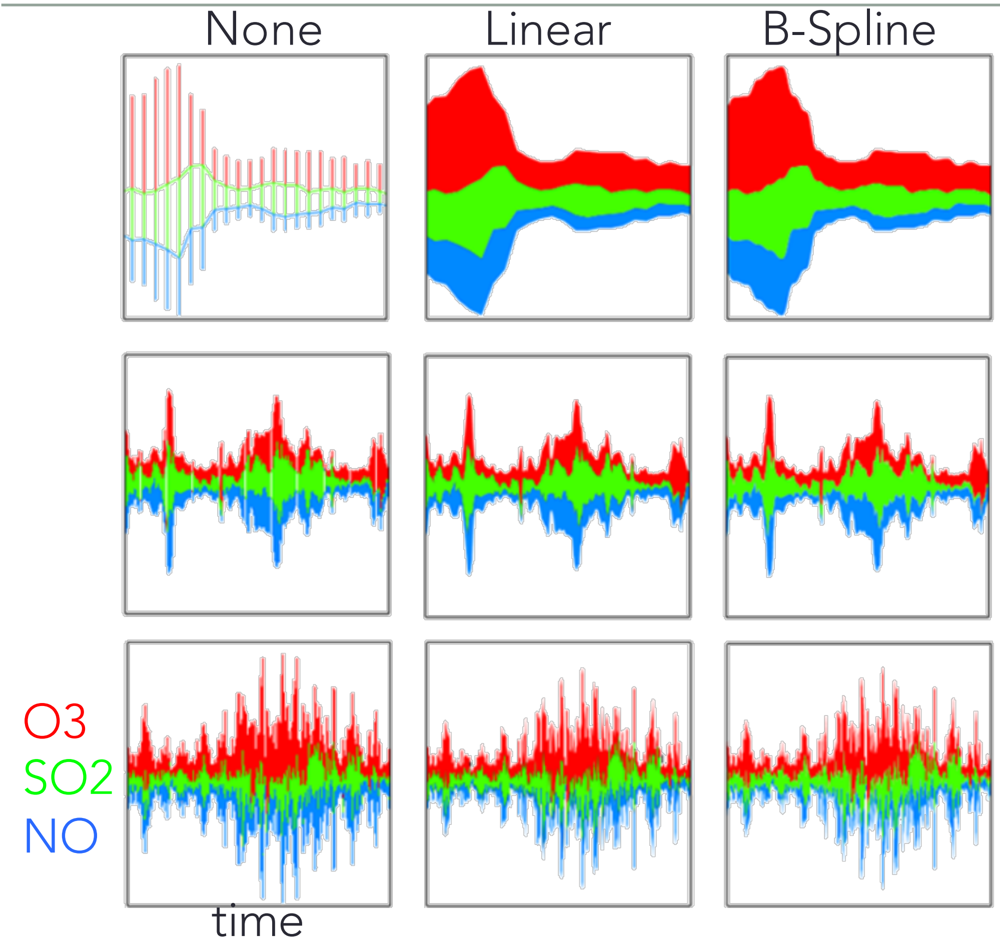
- Approximate computation
  - Until human can’t see the lexical level difference
    - pixel level JND
    - JPEG – compress until can’t tell pixel level difference
  - Until human can’t see the semantic difference
    - semantic level JND
    - correlation

3.3 How to do Visual-Centric Computation



- 2013 spring (Senior project)
- Visualize real-time air quality data in past 10 years
- 3 render strategies
- Switch between using hard cut-off

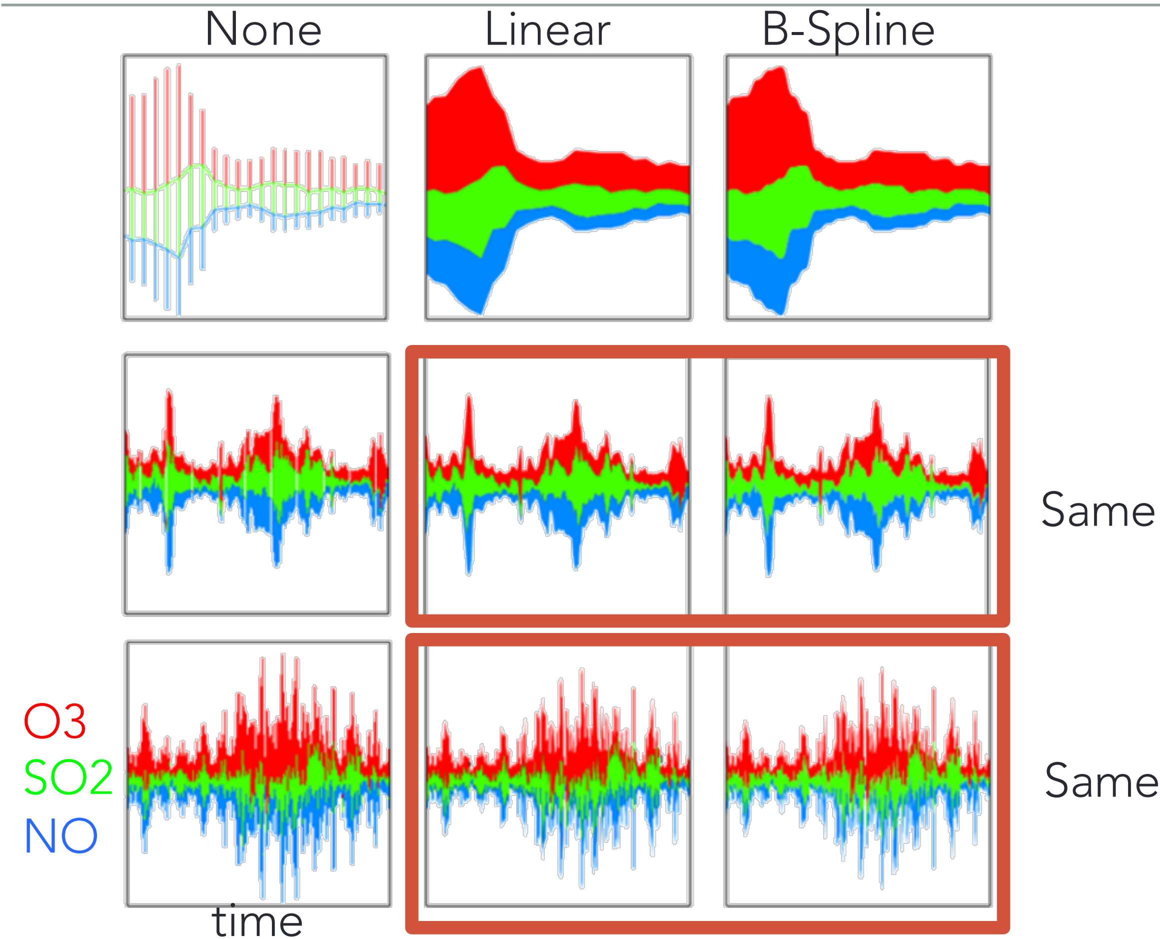
3.3 How to do Visual-Centric Computation



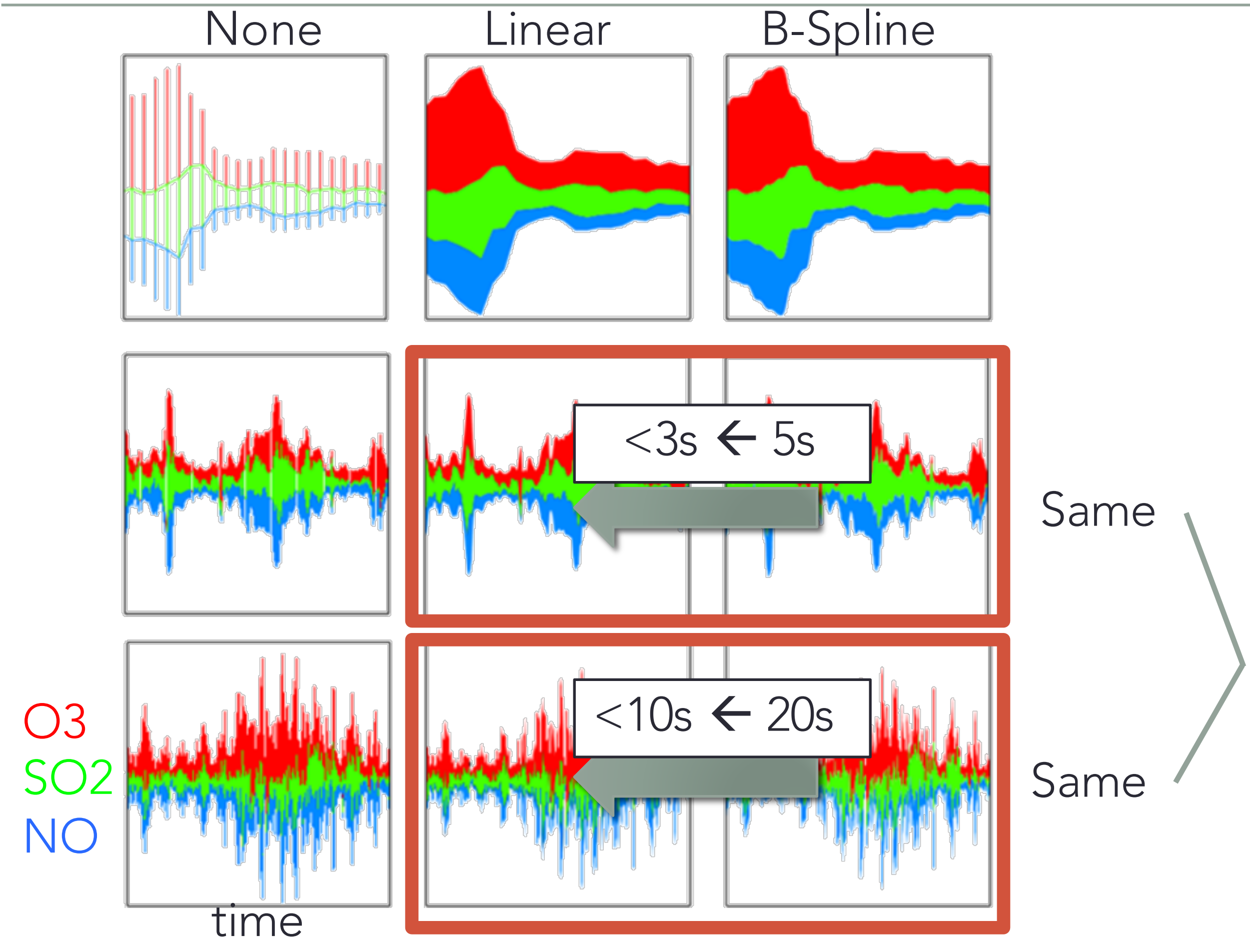
- 2013 spring (Senior project)
- Visualize real-time air quality data in past 10 years
- 3 render strategies
- Switch between using hard cut-off

- 2015 spring (New vision)
- Switch between using perceptual models

3.3 How to do Visual-Centric Computation

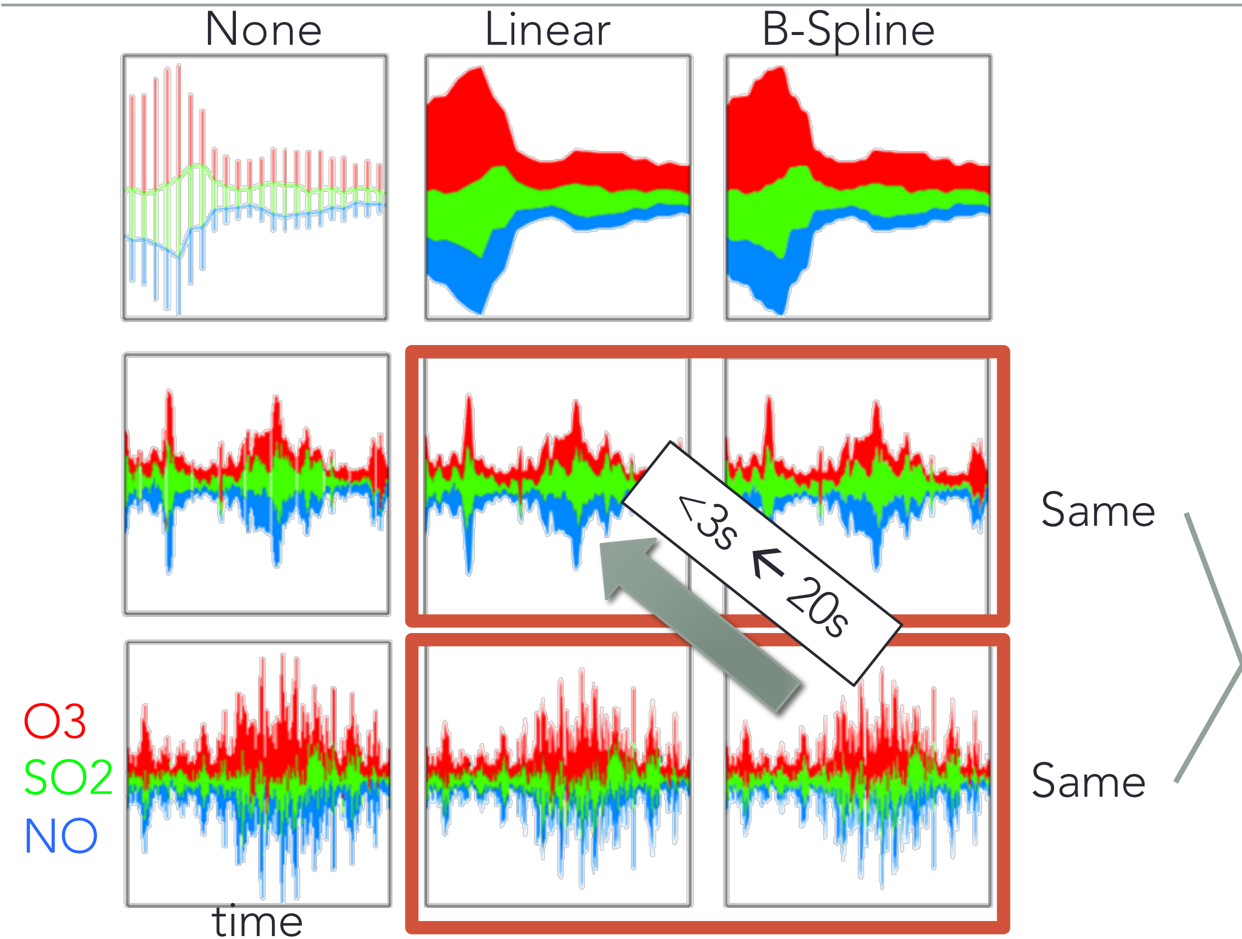


3.3 How to do Visual-Centric Computation



Use the faster strategy (linear)
Based pixel level JND

3.3 How to do Visual-Centric Computation



Use the faster strategy (linear)
Based on semantic level JND

3.3 How to do Visual-Centric Computation

- Visual-Centric Computation is a new way to think about general computation and visualization of big data
  - Using the new vision of my senior project as an example

Summary

1. Perceptual Model of Visualization
   - Weber's law holds for perception of correlation on 9 visualizations
   - Compare 9 visualizations using Weber models
2. Visual Feature
   - Why does the Weber's law work for correlation?
   - A visual feature is used as the substitute of correlation
3. Visual-Centric Computation
   - Apply perceptual models to computation
   - To speed up the result generating process; improve the understandability of the resulting visualization

Acknowledge

- Remco Chang, Lane Harrison
- Visual Analysis Lab at Tufts
- Steve Franconeri (Northwestern University)
- Ronald Rensink (University of British Columbia)
- Ruizhi Dai (Psychology Department)

From Perceptual Model Of Visualization To Visual-Centric Computation

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March 2015

Thanks!
Questions?

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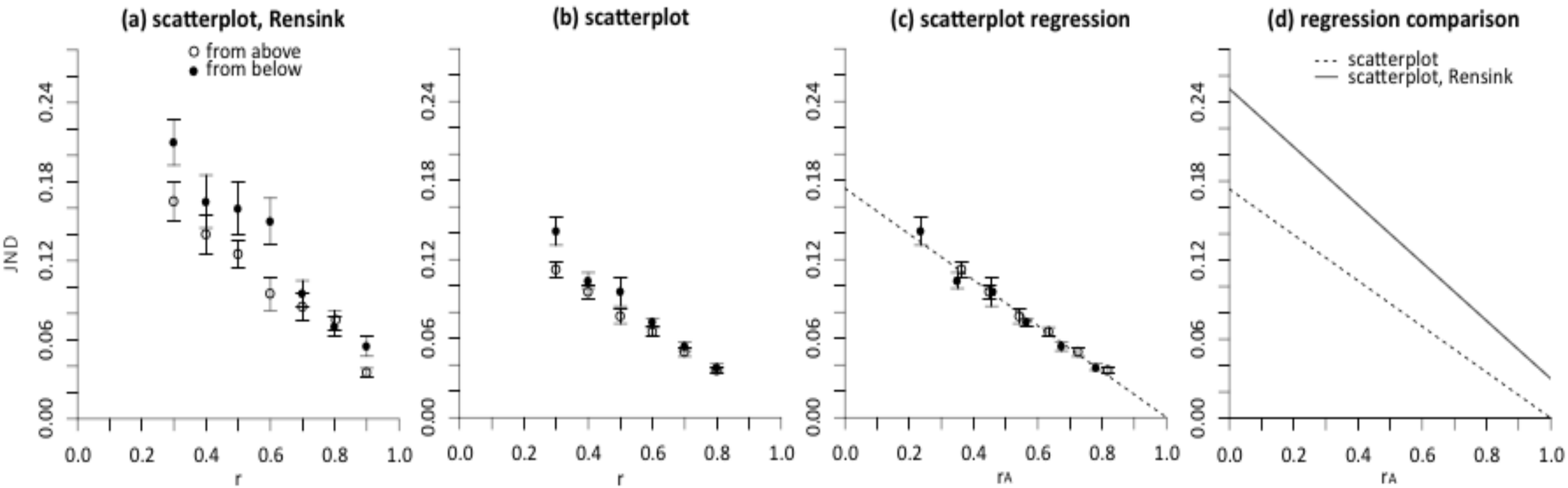
Tufts University
March 2015

Backup slides



(a) scatterplot, Rensink
(b) scatterplot
(c) scatterplot regression
(d) regression comparison
from above
from below
scatterplot
scatterplot, Rensink
JND
0.00
0.06
0.12
0.18
0.24
0.0
0.2
0.4
0.6
0.8
1.0
r
rA

The correlation coefficient of this starting dataset is then computed and noted as r_z. Then, each point (x_i, y_i) is transformed using the same transformation in [22]:

$$y'_i = \frac{\lambda x_i + (1-\lambda) y_i}{\sqrt{\lambda^2 + (1-\lambda)^2}} \tag{2}$$

where λ is defined as follows:

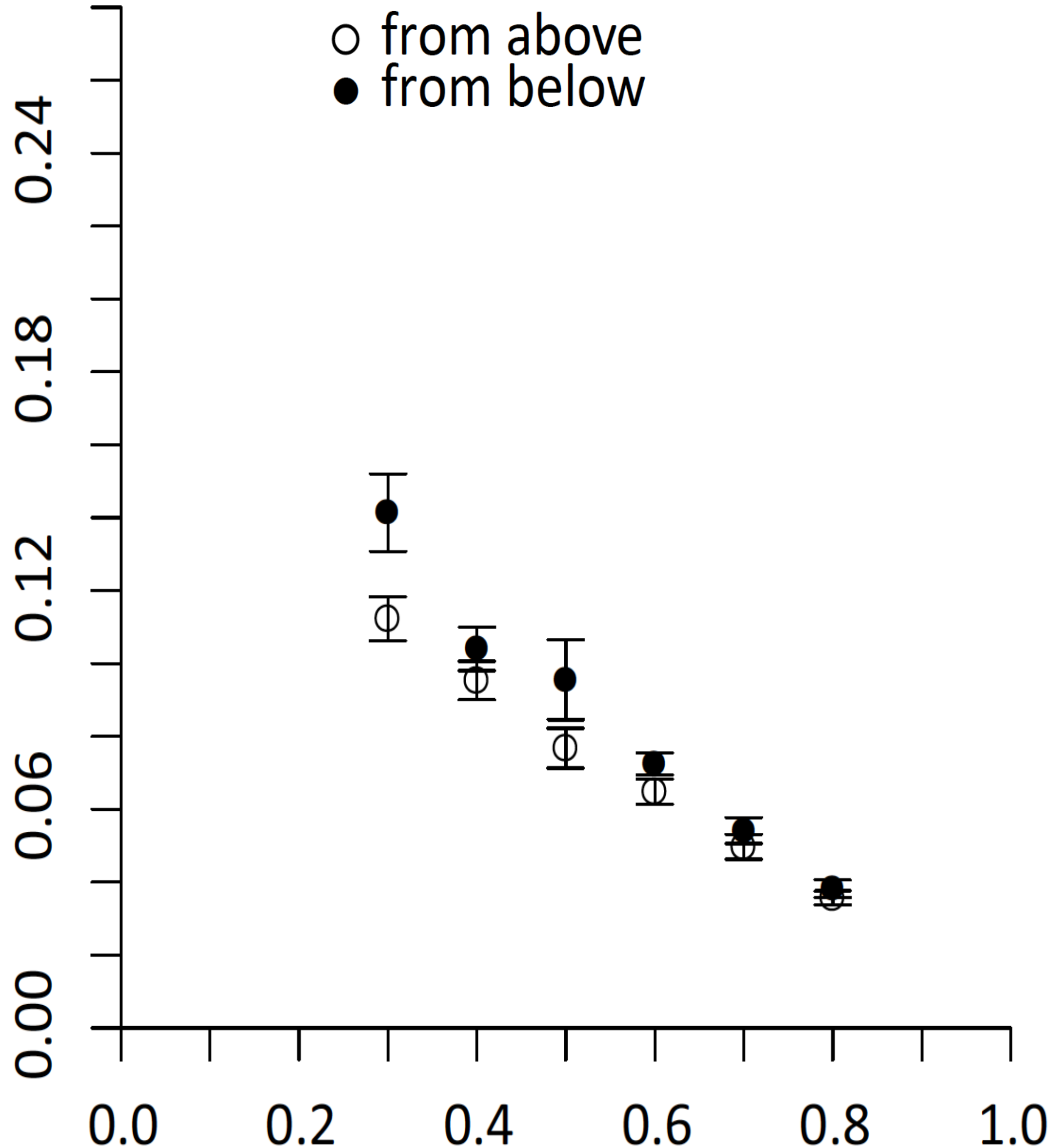
$$\lambda = \frac{(r_z - 1)(r^2 + r_z) + \sqrt{r^2(r_z^2 - 1)(r^2 - 1)}}{(r_z - 1)(2r^2 + r_z - 1)} \tag{3}$$

Data Generator

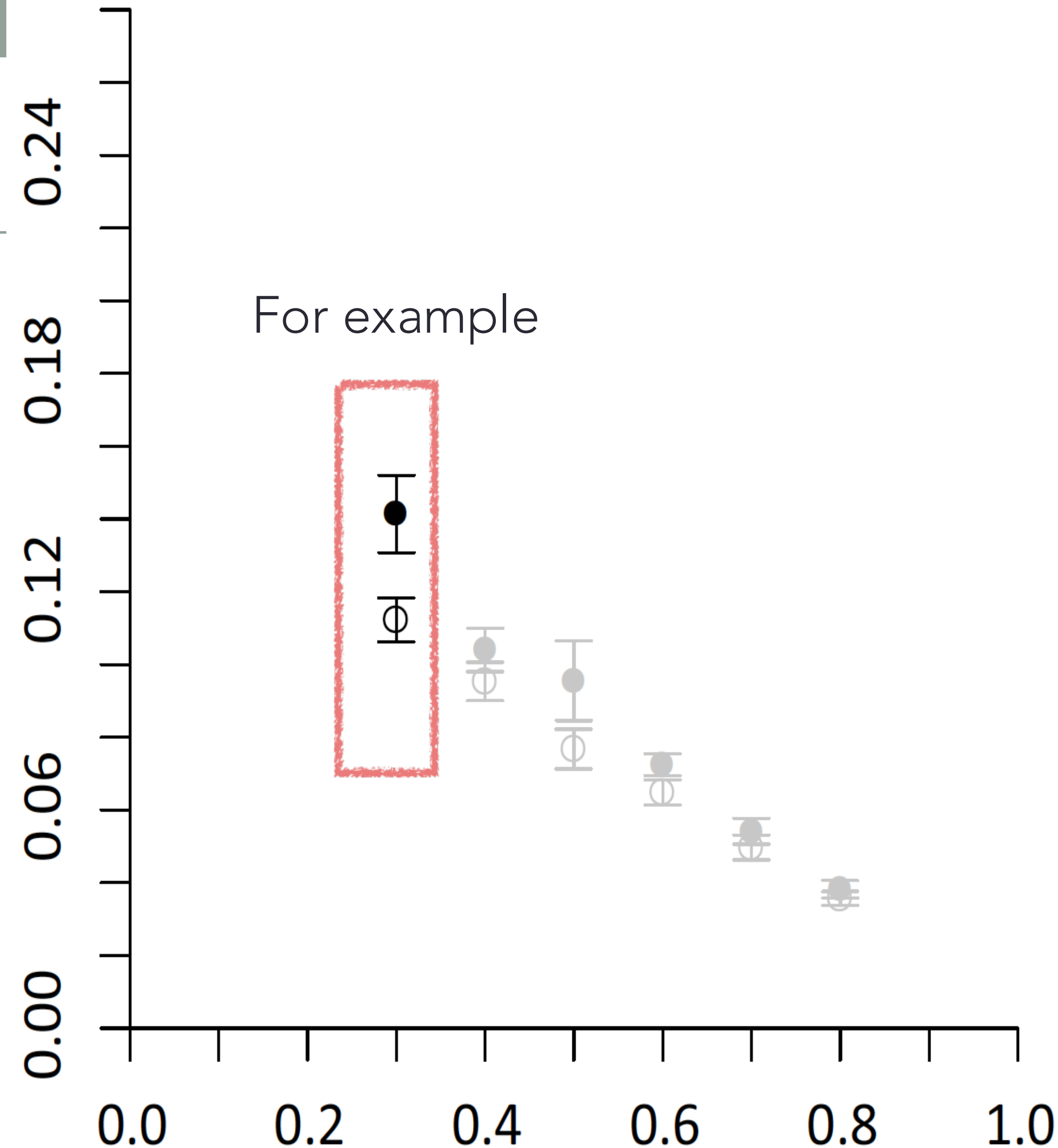
Four pages of proof please email me

Adjustment

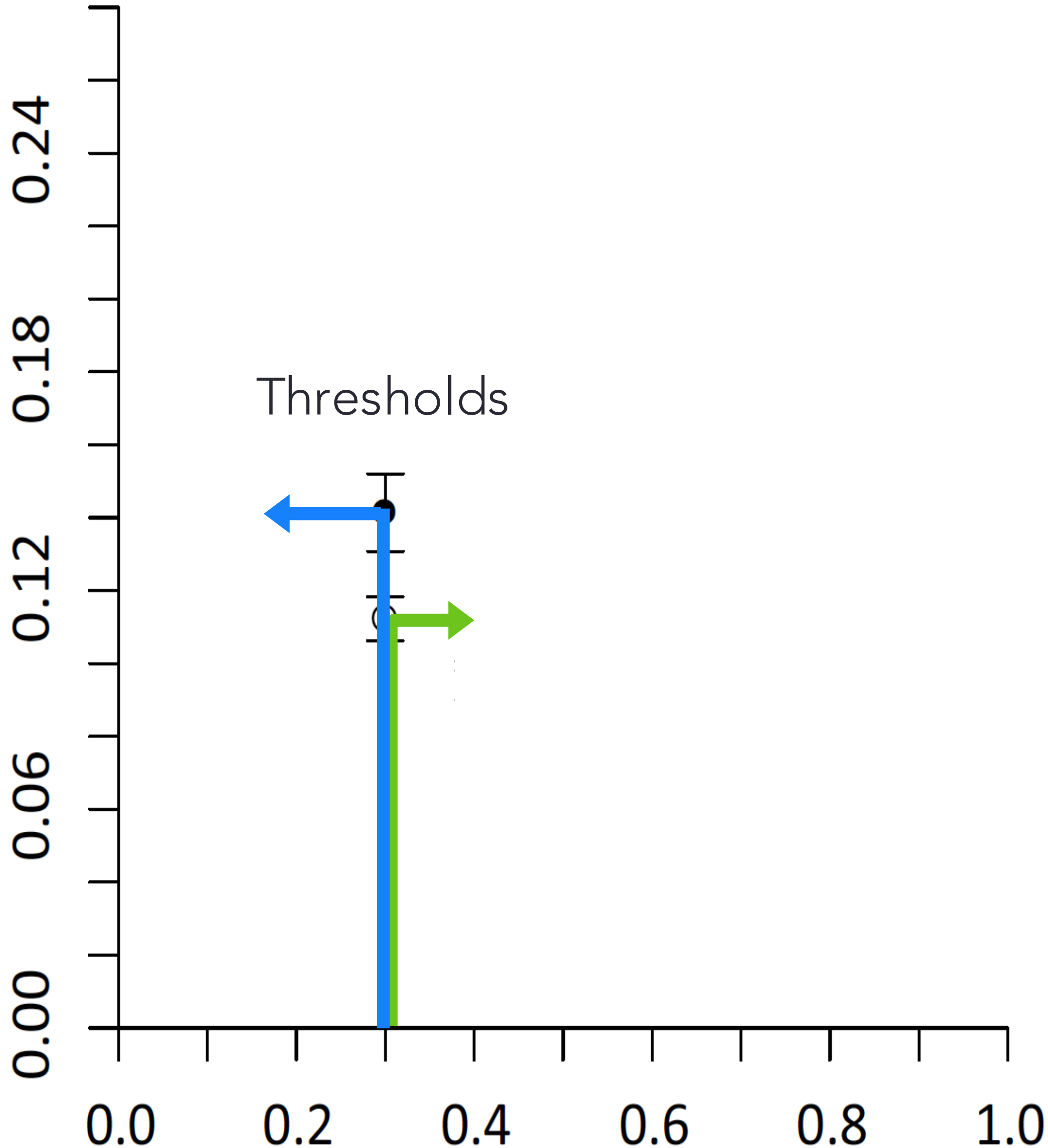
$$r_A = r \pm 0.5\, jnd(r)$$



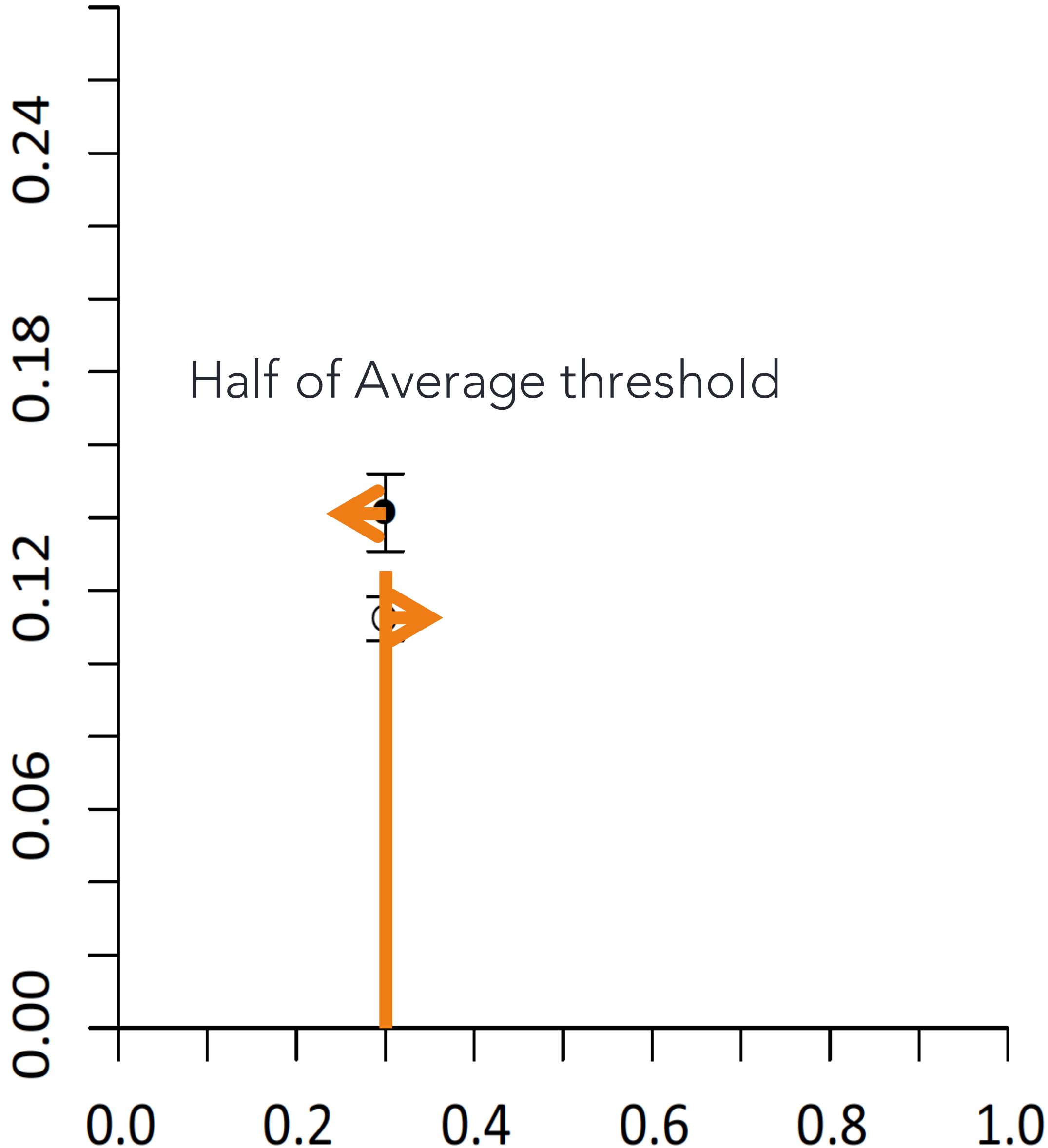
from above
from below
0.00
0.06
0.12
0.18
0.24
0.0
0.2
0.4
0.6
0.8
1.0

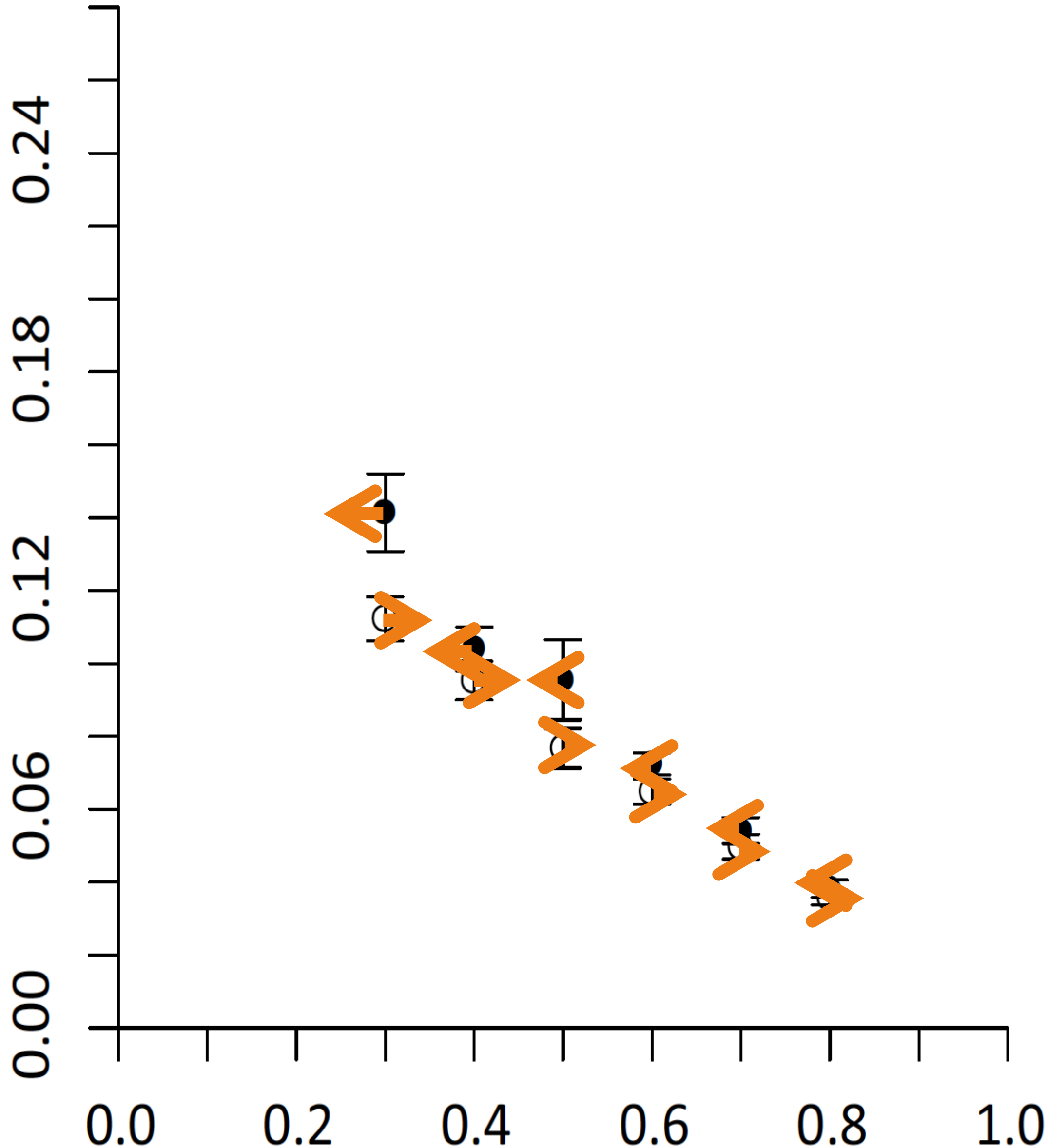


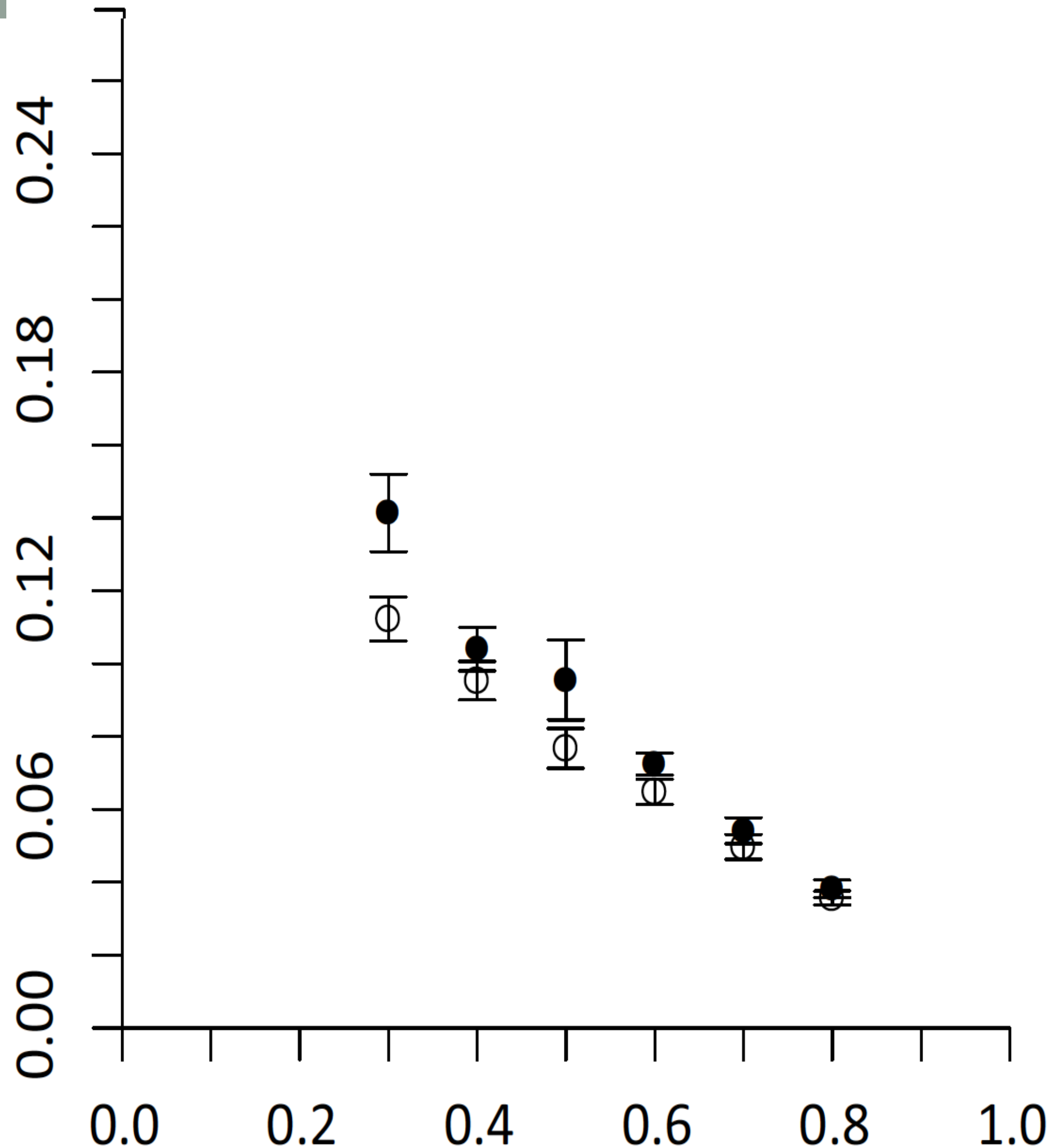
For example
0.00
0.06
0.12
0.18
0.24
0.0
0.2
0.4
0.6
0.8
1.0



Thresholds
0.00
0.06
0.12
0.18
0.24
0.0
0.2
0.4
0.6
0.8
1.0

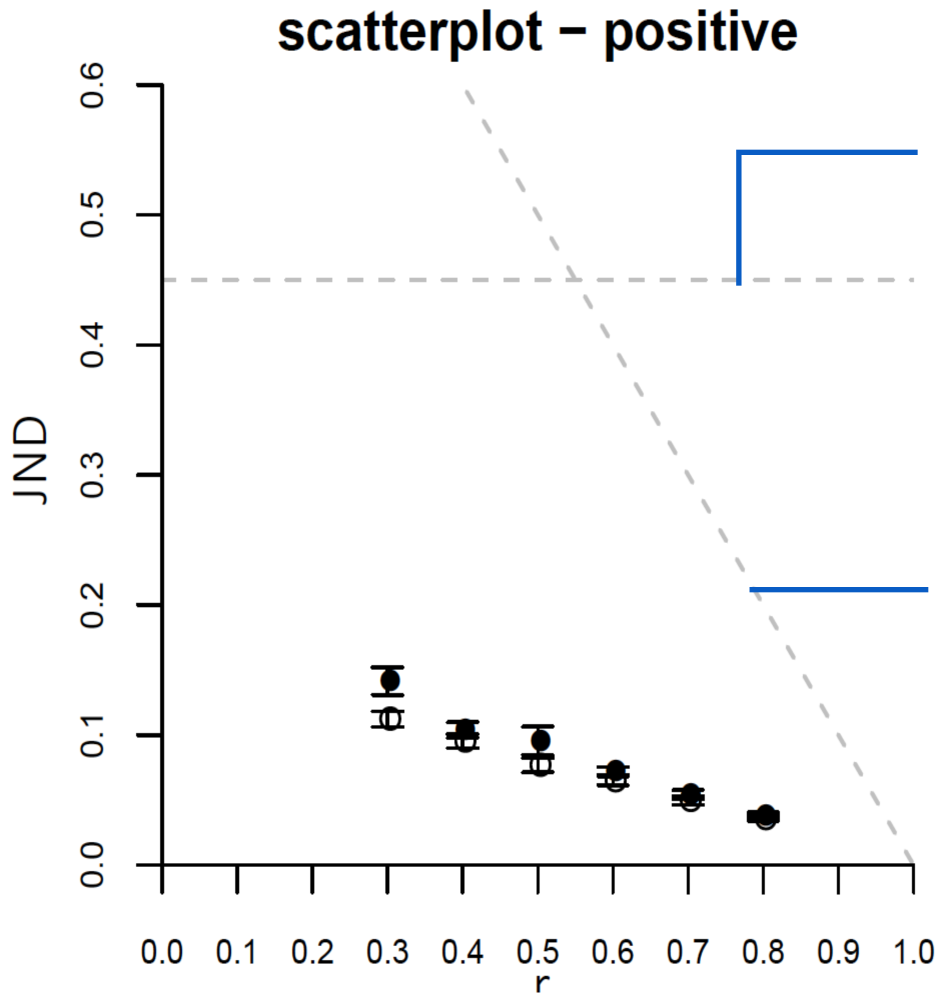

Half of Average threshold
0.00
0.06
0.12
0.18
0.24
0.0
0.2
0.4
0.6
0.8
1.0


0.00
0.06
0.12
0.18
0.24
0.0
0.2
0.4
0.6
0.8
1.0



0.24
0.18
0.12
0.06
0.00
0.0
0.2
0.4
0.6
0.8
1.0

Guessing Line
Ceiling Line



JND = 0.45

Guessling Line

Run 10,000 times of guessing

The average of JND, regardless of r

JND + r = 1

Ceiling Line